NOTES

ON

PLATE-GIRDER DESIGN

BY

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PREFACE

THE following Notes on Plate-Girder Design have been used by the author to give his civil engineering students the theoretical and practical information necessary to enable them to make a design and general detailed drawing for a through plate-girder railway bridge. The theory of the plate girder has been developed so that it may be applied to such structures for any duty. The notes may easily be given to the average class in one recitation per week for one semester of the college year. They have greatly facilitated the computation and drawing-room work with my own classes.

It is with the hope that they may be useful to other teachers, and engineers, that they are published.

C. W. HUDSON.

POLYTECHNIC INSTITUTE, BROOKLYN, N. Y.,
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NOTES ON PLATE-GIRDER DESIGN

ART. 1. STRESS DISTRIBUTION AND GENERAL

The theory of stress distribution for plate girders is the same as that for solid beams of uniform material, it being generally assumed that the common theory of flexure applies with sufficient accuracy to the built-up girder. This assumption is certainly not strictly true. The rivets connecting the various parts together cause a local and small irregularity of stress distribution; splices of the various parts, and non-prismatic form, produce quite marked irregularity of this distribution. The "Theory of Flexure" properly applied to plate-girder design, however, leads to a thoroughly good engineering structure, as an immense tonnage of such work constructed during the past forty years and doing excellent duty witnesses.

This wide use of the plate girder for service under greatly varying conditions has increased the knowledge of its capacity and given valuable practical information as to the design of certain of its features.

The requirements for its fabrication, shipment and erection have also affected its design. These requirements are not fixed in their nature, although in certain respects,

such as minimum thickness of material in web plates, and minimum edge distances for punching, they fix a limit to the size of certain parts.

The correct design of plate girders therefore requires theoretical knowledge and either practical experience or formulated rules, based on practical experience, and so carefully and closely drawn as to prevent a poor design.

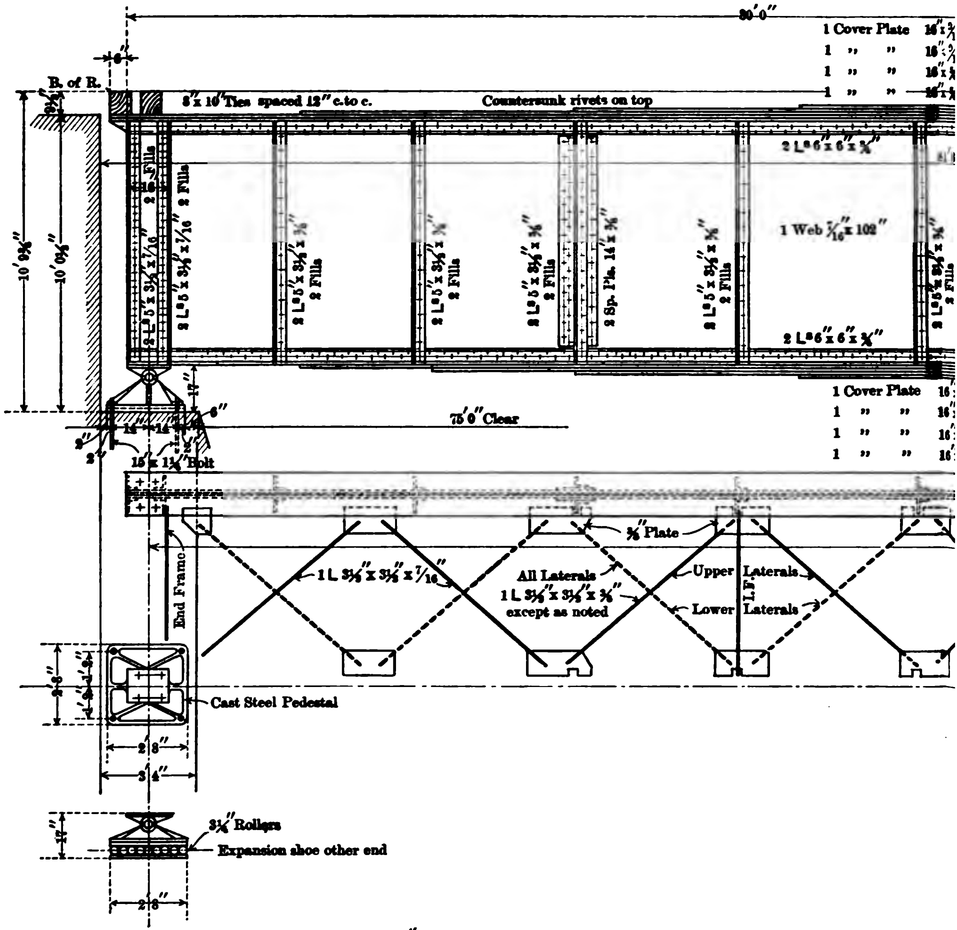
Plate girders are used for bridges anywhere from 18″ deep and 15′ long to 126″ deep and 130′ long, and even beyond these limits; they are also used in buildings and other important engineering work to a great extent. In general, the lower limit of their use is the I beam, which will furnish the proper strength at a less cost per pound of material; their upper limit is the truss, whose total cost is less than the heavier but cheaper per pound girder.

In order to design properly any structure it is necessary to understand the composition and relation of each to each of the various parts. The following drawings are meant to illustrate some of the most essential features of plate-girder construction. They are therefore not meant to be casually inspected, but thoroughly studied, and the function of the various parts in carrying a load to the supports, the makeup of the parts and their connection, each to each clearly understood.

Fig. 1*a* shows a single-track deck plate-girder bridge.

Fig. 1*b* shows a single-track through plate-girder bridge.

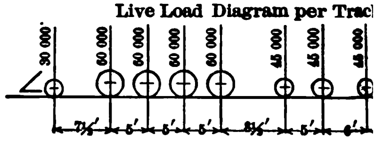
The computation of the stresses in any structure is the preliminary step in the design. The stresses themselves are a function of the weight of the structure, and hence an early estimate of the weight of each portion of the

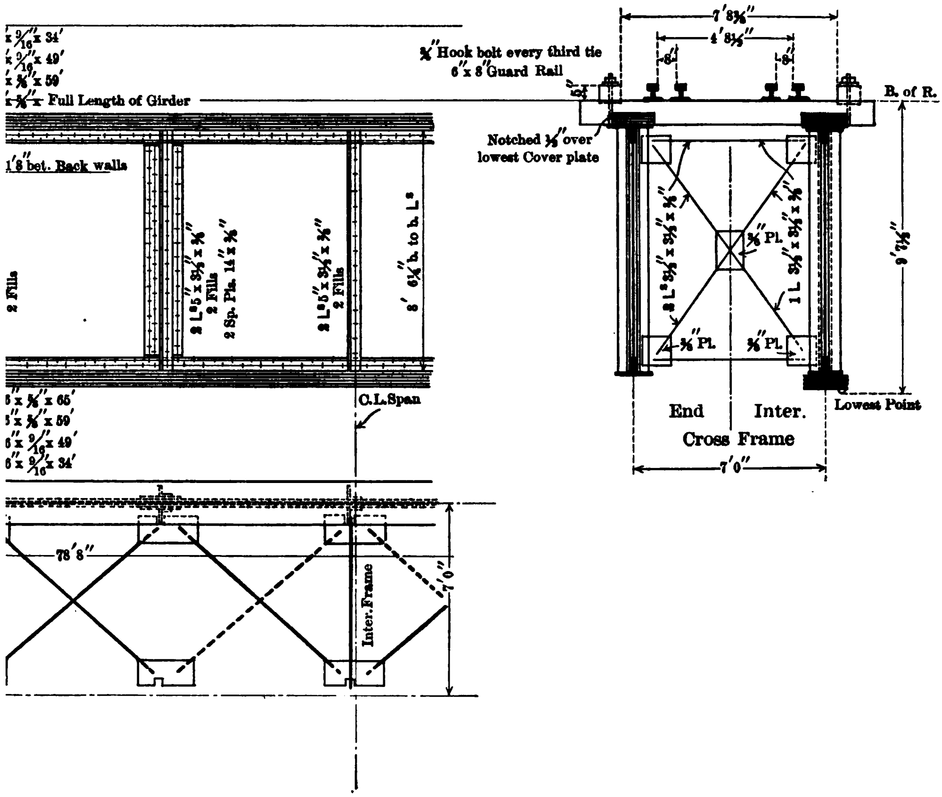


Estimated Weight 96,800 #

Dead Load 1700 # p. l. f.

Live Load Diagram per Trac





Bending, Live......	3,340,900	
Dead	658,100	
	3,990,000 + 8.5 = 470,500	
	at 10,000 = 47.05 sq. in.	
End shear, Live......	196,000	
Dead	34,000	
	230,000	

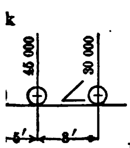
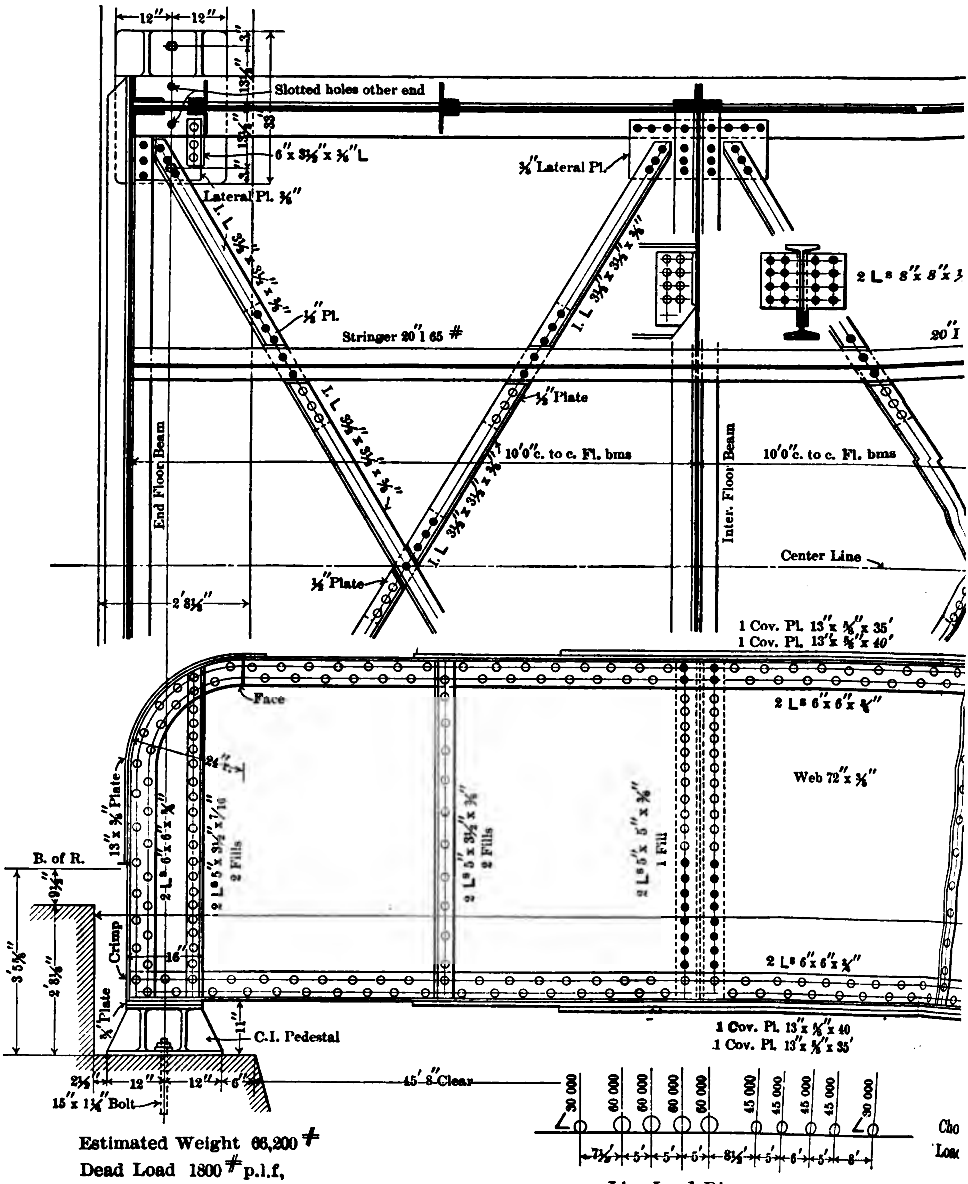


Fig. 1a.

STANDARD NO. 17

80'-0" OVER ALL, DECK PLATE GIRDER, S.T.
THE NATIONAL LINES OF MEXICO
Scale ¼" = 1'-0". April, 1907

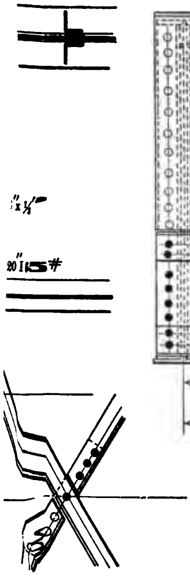
Boller & Hodge,
Consulting Engineers, New York

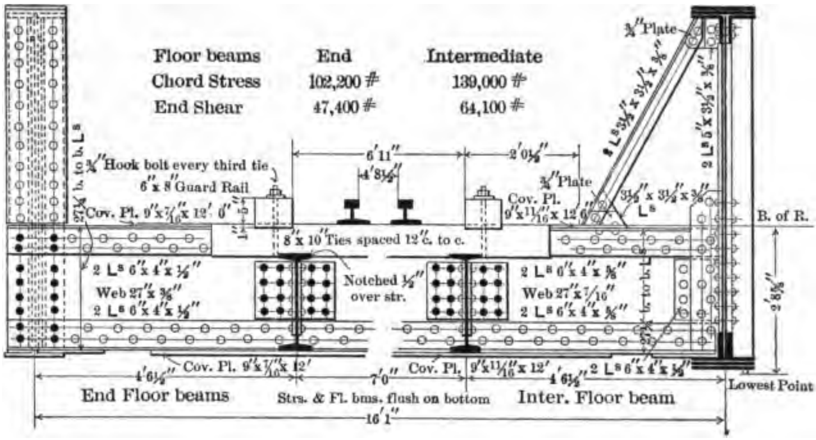


Estimated Weight 66,200 #
Dead Load 1800 # p.l.f,

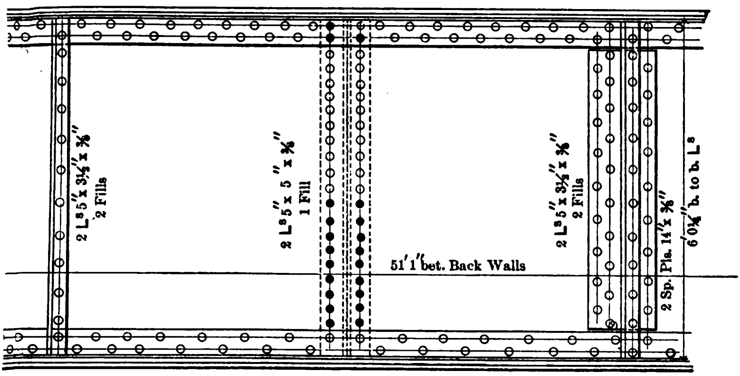
Live Load Diagram per Track

FIG. 1b.





Stringers	
Bending Moment	1,056,000 inch lbs.
End Shear	46,400 lbs.



Girder	
Chord Stress	272,800 #
Load on end shoe	158,600 #

Fig. 1b.

STANDARD NO. 12

50'-0" THROUGH PLATE GIRDER SPAN. THE NATIONAL LINES OF MEXICO

Scale ½" – 1'-0". April, 1907

BOLLER & HODGE,
Consulting Engineers, New York

structure is necessary. Many valuable compilations of weights of structures are to be had, but they are not usually accompanied by full data. Every engineer must be sure of the quantities which go into his structure, and they therefore should be estimated as the computations are carried along and all computations based on wrong assumed dead loads corrected.

The design of a structure should always begin with the part receiving the load and follow with each successive part in the path of the load as it travels to the support.

For example, in an ordinary through plate-girder railroad bridge, the design of the various parts should be made in the following order: Rails, ties, stringers, floor-beams, lateral bracing, and main girders.

In making a first estimate of the weight of any girder, compute the live-load stress at the center of the bottom flange and increase this for impact if any is to be added, add to this sum 15% for the stress due to the weight of the girder, then determine the net area of this member in square inches. This net area in square inches multiplied by 14 will give a weight per foot of girder which may be used as a first estimate.

This estimate is based on equal gross areas 15% greater than the net area of the bottom flange, for top flange, bottom flange, and web, and their sum increased 15% for details. An error of 1% in the total stress due to a wrong first assumed dead load should require a corrected dead load to be used.

It is very difficult to give a good general rule for estimating the weight of girders, as the weight obviously is a

function of the depth, loading, unit stresses, and many other things.

PROBLEMS

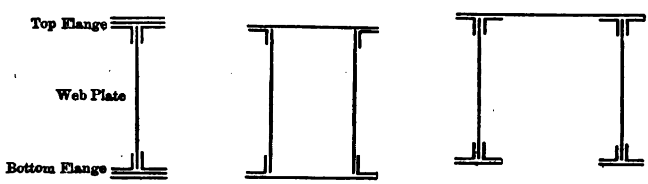
1*a*. Sketch the cross-section of a plate girder with and without cover plates, and write the name of each part on the sketches.

1*b*. Sketch the plan of single-track railroad deck plate-girder bridge and write the names of the various parts on the sketch.

1*c*. Sketch the plan of single-track railroad through plate-girder bridge, and write the names of the various parts in the sketch.

ART. 2. REQUIRED AREA OF CROSS-SECTION FOR THE FLANGES

The plate girder is a built-up structure, and may be made with a variety of cross-sections depending on the requirements of the case. The forms of cross-section most frequently used are those of *a*, *b* and *c* of Fig. 2.



a *b* *c*

FIG. 2.

The object of these forms being to take advantage of the known law of stress distribution in flexure.

The relation between the moment of the outer forces and inner stresses for homogeneous bodies subject to flexure is given from the formula

$$M = \frac{SI}{c},$$

in which M is the moment of the outer forces;
S is the unit stress in the extreme fiber of the cross-section under consideration;
I is the moment of inertia of the cross-section;
c is the distance from the neutral axis to the extreme fiber of the cross-section.

Any existing straight prismatic girder may be examined for intensity of flexural stress in a plane normal to the axis by means of this formula. The plate girder of prac-

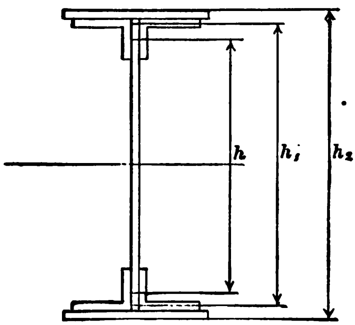


FIG. 2*d*.

tice is, however, far from the ideal prismatic bar of the Theory of Flexure. The application of the preceding formula to the design of girders is very tedious, and while it could be greatly facilitated by the use of tables of $\frac{I}{c}$ for a great variety of cases, the labor of its application to problems of design is not warranted.

The almost universal method of plate-girder flange design will now be developed, with the aid of the following nomenclature and Fig. 2*d*.

Let A be the net area of each flange;

h be the distance c. to c. of gravity of flanges;

S be the allowable unit stress for tension or compression produced by flexure;

h_1 be the depth of the web plate;

t be the thickness of the web plate;

h_2 be the depth out to out of flanges;

I_f be moment of inertia of each flange about its own horizontal axis.

Then the flexure formula becomes

$$M=\frac{SI}{c}=\frac{S}{\frac{h_2}{2}}\left[2\times A\times\left(\frac{h}{2}\right)^2+\frac{th_1{}^3}{12}+2I_f\right],$$

$$M=S\left[A\cdot h\cdot\frac{h}{h_2}+\frac{th_1}{6}\cdot h_1\cdot\frac{h_1}{h_2}+\frac{4I_f}{h_2}\right].$$

The right half of this equation represents the moment of the internal stresses; of the three terms which make up this half it may be said:

The term $\frac{4I_f}{h_2}$ is small in most cases, but for shallow girders it is relatively quite large, its omission from the expression requires the other terms to be larger.

The value of $\frac{h}{h_2}$ is always less than unity, and in shallow girders considerably less, frequently as much as 10%.

The value of $\frac{h_1}{h_2}$ is usually less than unity, for very shallow girders without cover plates it is often equal to unity, and for girders with two or more cover plates it may be as much as 10% less than unity.

If h, h_1, and h_2 be taken equal, and the term $\frac{4I_f}{h_2}$ be dropped the formula may be written:

$$M = S\left(A \cdot h + \frac{th_1}{6} \cdot h\right) = Sh\left(A + \frac{th_1}{6}\right).$$

That is, the approximate moment of the internal stresses equals the unit stress times the depth times the net area of one flange plus $\frac{1}{6}$ of the area of the web plate. Many engineers drop the $\frac{th_1}{6}$ from the expression, as its value is small and it is on the side of safety to do so. Others include it with the idea that as the web takes a part of the horizontal stresses these horizontal stresses should be considered in designing the flanges and web splices. Where stiffeners or splices are used on the web it is impossible to maintain the full web section, and the web available as flange is usually taken as $\frac{th_1}{8}$, which corresponds to a vertical rivet pitch of about 4″. The preceding formula should be written for the purpose of design:

$$A = \frac{M}{hs} - \frac{th_1}{6}, \quad . \quad . \quad (1) \quad \text{or} \quad A = \frac{M}{hs} - \frac{th_1}{8}. \quad . \quad . \quad (1')$$

In this formula $\frac{M}{h}$ is known as the flange stress, just as in the case of trusses the moment divided by the depth is the chord stress.

PROBLEMS

2a. Compute the net area required in the flanges of a plate girder 30′ long and 48″ deep out to out of flange angles, when carrying two loads, of 160,000 lbs. each, spaced 5′ from the center of the girder and a uniform load of 300 lbs. per linear foot. Assume a web plate of $48 \times \frac{3}{8}''$.

2b. Make up a section for the bottom flange of the girder of Prob. 2a, using only two angles.

2c. Make up a section for the bottom flange of the girder of Prob. 2a, using two cover plates and two angles.

ART. 3. THE DESIGN OF THE CROSS-SECTION OF THE FLANGES

Flanges of plate girders are generally composed of angles in pairs or angles in pairs and plates, as is shown in Fig. 2*a*, 2*b*, and 2*c*. The several parts are connected by rivets. The holes for the rivets are generally punched to a diameter $\frac{1}{16}''$ greater than that of the rivet, or to a diameter of $\frac{3}{16}''$ less and subsequently reamed to $\frac{1}{16}''$ greater, or, as in the best class of railway work, drilled to a diameter $\frac{1}{16}''$ greater than that of the rivet. It is customary in designing tension members to allow for a hole $\frac{1}{8}''$ greater in diameter than that of the rivet, that is, for a $\frac{7}{8}''$ rivet a hole 1″ in diameter should be deducted. The number of holes to be deducted from any tension flange depends on the number of rows of rivets and the spacing of the rivets in the rows. Much might be written on this point, but here only little will be given.

For the flange shown in *a* in section, the views of *c* and *d* are longitudinal developments of one of the angles showing common methods of grouping the rivets.

It is clear that a symmetrical arrangement of rivets such as shown in c is better than that of d, for the center of gravity of the net area through x–x and y–y of c lies in the root of the angle, while for the corresponding sec-

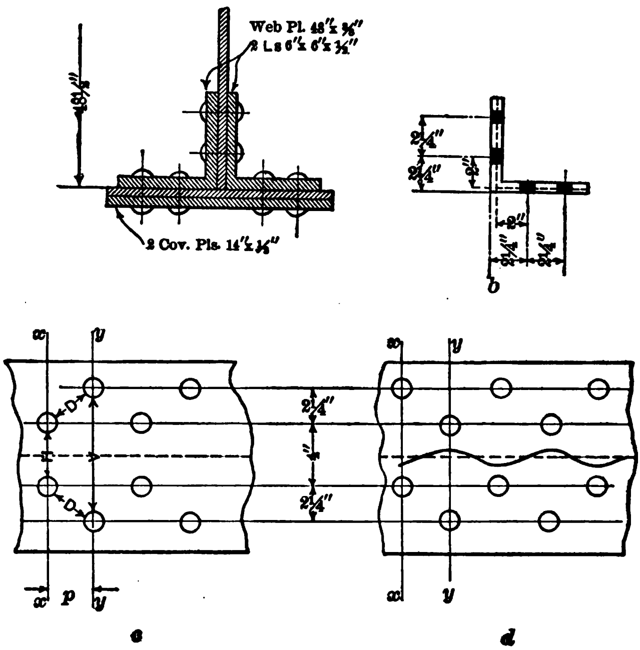


Fig. 3.

tions of d it is first to the right of the root and then to the left, as indicated. The arrangement at d is sometimes chosen because it permits a little more freedom in driving one of the inner lines of rivets. Care must be taken in locating the sections x and y so that $2D+H \gtreqless V$; for the case selected the longitudinal pitch p cannot be less than $\sqrt{3.25^2-2.25^2}=\sqrt{\frac{88}{16}}=\frac{1}{4}\sqrt{88}=2.35''$. In fact the stresses

passing through the space marked *D* are considerably bent and thereby increased, and therefore the distance *p* for a good stagger for this case should be 3″. It should be noted that most girders have an excess of strength except at the point of maximum bending moment and at the ends of cover plates, and hence care in staggering the rivets need only be exercised at these points.

The entire flange stress is developed in small increments by the web and transmitted to the flange by the rows of rivets connecting the vertical legs of the angles to the web. It is clear that these angles then should comprise a considerable part of the total flange area—some engineers require 50%, others permit as little as 33⅓%. The girder which has a flange stress developed in a short distance requires heavier angles than one in which the stress is developed in a great distance. The foregoing provision for maintaining net section applies particularly to the tension flange. The compression flange is usually made equal in gross area to the tension flange.

The compression flange is in somewhat the condition of a column as far as liability to failure in a sidewise direction is concerned. For girders with a constant top flange section the maximum unit stress occurs only at the point of maximum moment, for a flange with cover plates; that is, for a flange which varies closely as the flange stress, the unit stress is nearly constant throughout. The web stiffeners, if any are used, give considerable lateral stiffness to the top flange, reaching and connecting, as they do for half their length to material in tension. The sketch of Fig. 3*e* will help make this point clear. The tension flange

is held in line by virtue of its stress; any tendency to sidewise deflection of the compression flange is resisted by the tension flange if stiffeners are used.

If the unit stress for static loads in tension is 16,000 lbs. per square inch, then $16{,}000 - 70\frac{l}{r}$ is a corresponding unit stress for a compression member, in which the quantities need no definition. If it be assumed that a girder flange

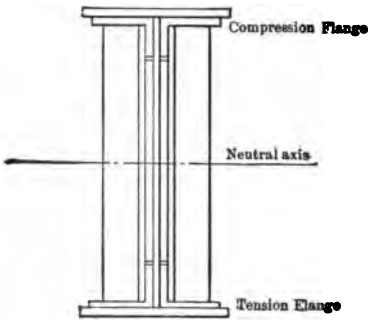


FIG. 3e.

is similar to a column in its action and that its section is a rectangle whose width $=b$, then

$$\left(16{,}000 - 70\cdot\frac{l}{r}\right) = \left(16{,}000 - 70\cdot\frac{l}{b}\sqrt{\frac{1}{12}}\right) = 16{,}000 - 242\frac{l}{b},$$

or

$$16{,}000 - 240\frac{l}{b}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

gives a compressive unit stress for a girder flange corresponding to 16,000 lbs. per square inch in tension. As the stress in a girder flange is only a maximum for a

small part of its length a formula for the safe compressive and unit stress

$$P = 16{,}000 - 200\frac{l}{b}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2')$$

may be used.

Experience has shown that no material should be used which is less in thickness than $\frac{1}{16}$ of the distance between the rivets in the direction of the action of the stress, $\frac{1}{8}$ of the distance from the center of a rivet to the edge of the piece at right angles to the line of stress, or that any angle leg when used alone in a girder flange shall be longer than 12 times its thickness, otherwise the girder flange may fail in detail rather than as a whole.

For simplicity and ease of construction the compression flange should not be made of greater section than the tension flange, therefore the compression flange must be supported in a sidewise direction at frequent intervals, which, from the preceding formula, will be about every ten times the flange width. It is customary to consider that the rivets completely fill the rivet holes in the compression flange and that the full gross section of the flange may be assumed to act to resist compression. This assumption with regard to the rivets always filling the holes is open to serious question, and if they do fill the holes they do not offer the same resistance to lateral deformation which takes place under compression as the unpunched material. However, it is fair to assume that they partially make up the punched-out material. The formula for the unit stress for the compression flange is believed to be severe enough to allow it to be applied to the gross area

of the flange. The depth h (called the effective depth) of formula (1) is obtained by taking the gross area of both flanges in computing the location of the center of gravity of the flanges.

It should be borne in mind that if there is no lateral deflection of the top flange that its maximum unit stress depends entirely on $\frac{I}{c}$, the section modulus, as it is sometimes called. While rivet holes affect the position of the neutral axis for the sections in which they occur, probably two-thirds or one-half of the length of a girder will be undiminished by holes, hence in applying the formula $M=\frac{SI}{c}$ to check the results of formula (1) it will be best to determine the position of the neutral axis from the gross section, and find the moment of inertia of the net section of the entire girder section for I in the formula.

PROBLEMS

3*a*. Compute the probable maximum unit stress in the tension flange of the girder of Prob. 2*b*, by means of the formula $M=\frac{SI}{c}$.

3*b*. Compute the probable maximum unit stress in the tension flange of the girder of Prob. 2*c*, by means of the formula $M=\frac{SI}{c}$.

3*c*. Compute the probable maximum unit stress in the compression flanges of Probs. 3*a* and 3*b*, by means of the formula $M=\frac{SI}{c}$.

ART. 4. LENGTHS OF COVER PLATES

The plate girder, being a composite structure, may easily be constructed so that its cross-section may vary approximately as the moments and shears require. The full flange section being required only where the moment is a maximum, a method of determining where the parts may be omitted when no longer required is necessary to economically design the girder. In general the cover plates are the only parts of the flange which do not extend the full length. Two methods will be developed for finding where cover plates (or any other part of the flange) may be omitted.

The first: For girders which carry a uniform load, or a load which may be closely represented by a uniform load. Deck plate girders for railway bridges may be included in this classification.

In Art. 2 the approximate moment of the internal stresses at any section $=Sh\times\left(A+\frac{th_1}{6}\right)$ in which $\left(A+\frac{th_1}{6}\right)$ is the net flange area, designating this area by a for simplicity, the moment of the internal stresses becomes Sha.

Let $w=$ the uniform load per foot of girder, or for locomotive loading the uniform load which would produce the same end shear, then the bending moment at any point distant x from the end $=\frac{wlx}{2}-\frac{wx^2}{2}$, and this must equal the moment of the internal stresses.

$$\therefore \qquad \frac{wlx}{2}-\frac{wx^2}{2}=Sha,$$

if it is desired to find the location of the point n, where the first cover plate must begin, substitute for S, h, and a their value for the portion of the girder between the end and n and solve for x. It is more cnovenient in practice to have the formula in such form as to give the length of cover plate direct. For this puprose

Let c = the theoretical length of any cover plate $= l - 2x$;

C = the practical length of any cover plate $= c + y$ (y = from 2 to 5 feet) the additional length y being required for locating a few rivets, so that the plate may be capable of taking stress where it is theoretically required.

FIG. 4*a*.

From the foregoing relation between the moments of the outer and inner forces may be written

$$x^2 - lx + \frac{l^2}{4} = \frac{l^2}{4} - \frac{2Sha}{w} \quad \text{and from this} \quad x = \frac{l}{2} - \sqrt{\frac{l^2}{4} - \frac{2Sha}{w}}.$$

$$\therefore \quad C = y + c = y + l - 2x$$

$$= y + l - 2\left(\frac{l}{2} - \sqrt{\frac{l^2}{4} - \frac{2Sha}{w}}\right) = y + 2\sqrt{\frac{l^2}{4} - \frac{2Sha}{w}}. \quad . \quad (3)$$

To use the formula to find the length of the second plate, take a and h for the portion of the girder between n and o.

Another simple formula by the author for finding the lengths of cover plates for this class of loading is given in Eng. News, XXXII, page 278, the issue of Oct. 4, 1894.

The second: For girders carrying loadings which may not be represented by a uniform load. Girders of this class receive their loads through other girders or columns at definite points. The main girders of through plate girder spans with floorbeams are common examples of this class.

As before the resisting moment $=Sha$, where

a = area of flange at end of any cover;

h = depth c. to c. of gravity of flanges at end of any cover;

S = unit stress.

The bending moment M at any point along the length of the beam and distant x from the left end, where P_1, P_2, P_3, etc., of Fig. 4b are the concentrated loads, w the

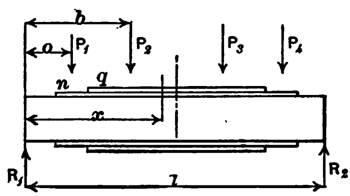


FIG. 4b.

uniform load per foot of length, and R_1 the left hand reaction, is given from

$$M = R_1x - wx\cdot\frac{x}{2} - P_1(x-0) - P_2(x-b) - \text{etc.}$$

Since the moment of the external forces = moment of internal stresses

$$R_1 x - \frac{wx^2}{2} - P_1(x-0) - P_2(x-b) + \text{etc.} = Sha, \quad . \quad (4)$$

in which every quantity is known except x.

To apply this to any point n between the first concentrated and the end the above becomes

$$R_1 x - \frac{wx^2}{2} = Sha, \quad . \quad . \quad . \quad . \quad . \quad (4')$$

in which R_1 and Sha are known, and the solution of which is a very simple matter. It should be noted that the proper position for the live load is that which makes R_1 a maximum.

To apply it to any point q between two loading points the formula becomes,

$$R_1 o + (R_1 - P_1)(x - o) - \frac{wx^2}{2} = Sha. \quad . \quad . \quad .(4'')$$

The position of the live load must be taken, first, so as to make the bending moment at P_1 a maximum; second, so as to make the bending moment at P_2 a maximum.

This gives two equations of the form of (4″) each of which must be solved for x.

The value of x, which is least, i.e., the one requiring the longest cover, is to be taken.

The lengths of cover plates are readily found by a graphic method which needs no explanation beyond the following sketch: 4*c*. The full line is drawn to represent

the resisting moment of the various portions, and the dotted line the moments of the outer forces.

The application of formulas (3) and (4) will now be made to finding the lengths of cover plates; for this purpose assume a girder 43′ long c. to c. end bearings, $60\frac{1}{2}''$

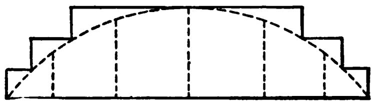


Fig. 4c.

deep out to out of flange angles and of the following composition:

1 web plate $60 \times \frac{3}{8}'' = 22.50$ sq.in. $\times \frac{1}{8}$ $= 2.81$ sq.ins.
2 top angles $6 \times 6 \times 19.6 = 11.52 + 2.81$ $= 14.33$ sq.ins.
1 top plate $14 \times \frac{1}{2} = 7.00 + 14.33 =$ $= 21.33$ sq.ins.
1 top plate $14 \times \frac{1}{2} = 7.00 + 21.33 =$ $= 28.33$ sq.ins.
2 bottom angles $6 \times 6 \times 19.6 = 11.52 - 2.00 = 9.52 + 2.81$ $= 12.33$ sq.ins.
1 bottom plate $14 \times \frac{1}{2} = 7.00 - 1.00 = 6.00 + 9.52 + 2.81 =$ 18.33 sq.ins.
1 bottom plate $14 \times \frac{1}{2} = 7.00 - 1.00 = 6.00 + 15.52 + 2.81 =$ 24.33 sq.ins.

The girder will first be assumed to act as in a single-track deck railway bridge, using formula (3),
Maximum bending moment $= 1{,}938{,}410$ ft.lbs.

Maximum end shear $= 193{,}500$ lbs. $\left(w = \frac{193{,}500}{21.5} = 9000\right)$.

$$a = 12.33,\ 18.33,\ \text{and}\ 24.33\ \text{sq.in.}$$
$$h = 4.77,\ 4.85,\ \text{and}\ 4.93\ \text{ft.}$$

For first cover $C_1 = y + 2\sqrt{\frac{(43)^2}{4} - \frac{2 \times 12.33 \times 4.77 \times 16{,}000}{9000}}$

$$= y + 2\sqrt{462.2 - 209.1} = y + 2\sqrt{253.1}$$

$$= 2 + 2 \times 16 = 34' \text{ long.}$$

For second cover $C_2 = y + 2\sqrt{462.2 - 2 \times \frac{1833}{9000} \times 4.85 \times 16{,}000}$

$$= y \times 2\sqrt{462.2 - 316.1} = y + 2\sqrt{146.1}$$

$$= 2 + 24 = 26'.$$

The girder will now be assumed to have floorbeams attached to it as indicated in Fig. 4*d*. The reaction R_1, when the moment at P_1 is a maximum, is 136,500 lbs. and the own weight of the girder 300 lbs. per linear foot.

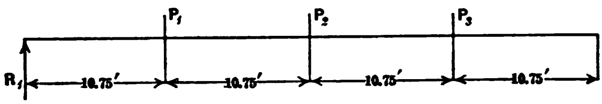


FIG. 4*d*.

Then for the first cover

$$136{,}500x - 300\frac{x^2}{2} = 12.33 \times 4.77 \times 16{,}000 = 941{,}000.$$

$$910x - x^2 = 6273 \quad \text{or} \quad x^2 - 910x + 207025 = -628 + 207025$$

$$x = 45.5 - 38 = 7.5.$$

The length of the first cover $C_1 = 2 + (43 - 15) = 30'$.

For the second cover

$$136{,}500x - 150x^2 = 18.33 \times 4.85 \times 16{,}000 = 1{,}422{,}400.$$

$$x^2 - 910x = 9483;$$

$$x = 455 - 445 = 10.$$

The length of the second cover $C_2 = 2 + (43 - 20) = 25'$.

PROBLEMS

4a. Let each of the flanges of the girder of this Art. consist of

2 angles $6 \times 6 \times 17.2$ lbs. $= 10.12$ sq.ins.
1 cover plate $14 \times \frac{3}{8}$ " $= 5.25$ "
1 cover plate $14 \times \frac{3}{8}$ " $= 5.25$ "
1 cover plate $14 \times \frac{3}{8}$ " $= 5.25$ "
and $\frac{1}{8}$ of the web plate $60 \times \frac{3}{8}$.

Find the lengths of the cover plates when it acts as a part of a deck railway bridge.

4b. Find the lengths of the cover plates when the girder of Prob. 4*a* acts as a part of a through bridge. The concentrated load $P_1 = 105{,}000$ lbs. The loads being spaced as shown in Fig. 4*d*.

ART. 5. RIVET SPACING IN GIRDER FLANGES

The connection between the web and flanges of girders, as in other composite structures, is made by means of rivets. These rivets are spaced with reference to the horizontal component of the stress in the flange, for at the extreme fiber the direction of the stress is horizontal and the maximum shearing unit stresses in the flanges are very small, and hence the horizontal component is the only stress of importance.

The one important exception to the foregoing is where the girder load is applied to one of the flanges, here the rivets have to transmit the loading which the girder carries together with the horizontal stress increments between the flanges and web. The exceptional case will receive special consideration.

The moment of the external forces at any point in a girder equals the moment of the internal stresses, there-

fore between any two points it is essential that there be enough rivets to properly develop the stresses produced by the maximum increase in moment between the points. The increment in the moment is constantly varying throughout the girder length. The general equation for moment for a simple girder as given in Art. 4 is

$$M = R_1 x - \frac{wx^2}{2} - P_1(x - 0) - P_2(x - b) - \text{etc.},$$

The derivative of M with respect to x is,

$$\frac{dM}{dx} = R_1 - wx - P_1 - P_2 - \text{etc., and is the shear.}$$

It is therefore seen that the greatest increase in the bending moment occurs when the shear has the greatest possible value.

The increase in bending moment between two points so close together that the load between them may be neglected, is the shear multiplied by the distance between the points; the increase in flange stress is this increase in moment divided by the depth.

Let V = the maximum shear at any point on the girder;

h = the effective depth, i.e., the depth c. to c. of gravity of flange, in inches;

R = the least value of the rivet to resist either crushing or shear, and

p = the space between two adjacent rivets in inches.

Then $\frac{V \times 1}{h}$ = the maximum increase in flange stress in a space of 1 inch . . . . (*a*)

$\frac{R}{p}$ = stress per inch of girder length carried by the rivets . . . . (*b*)

For the proper degree of strength (*a*) should equal (*b*).

$$\therefore \qquad \frac{V}{h}=\frac{R}{p} \quad \text{or} \quad p=\frac{Rh}{V} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In order to show the application of (5) to determining rivet pitch: Let Fig. 5*a* represent a portion of the girder

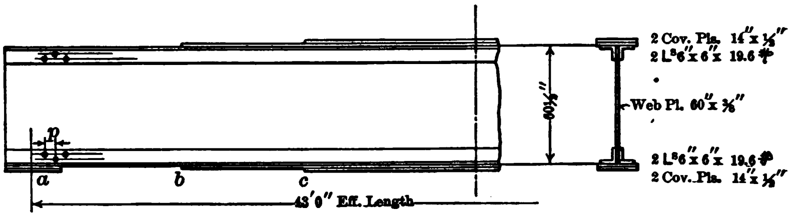


FIG. 5*a*.

used to illustrate the method of finding lengths of cover plates in Art. 4.

The shear at $a = 193{,}500$, at $b = 153{,}000$, and at $c = 118{,}000$.

The value of a rivet in bearing $= 24{,}000$, and in shear 12,000 lbs. per sq.in.

The pitch for ⅞″ rivets, at (*a*) =

$$p=\frac{4.77 \times 12 \times 7876}{193{,}500}=2.34''$$

provided the flange angles are designed to carry the entire flange stress at this point. If, however, as was assumed in finding the length of the cover plates, the portion of the web used as flange area is 2.81 sq.in., then the pitch $=\frac{2.34 \times 12.33}{(12.33-2.81)}=\frac{2.34 \times 12.33}{9.52}=3.03''$. The pitch of 2.34″ would require that two lines of rivets be used, as good construction requires that the rivets be not closer

than 3″. It is also customary to use two rows of rivets to connect a 6″ angle leg to the web.

If the load is applied directly to the girder flange then the previous computation needs modification for the rivet pitch in the vertical legs of the angles of the loaded flange. Let Fig. 5*b* represent a part of the top flange of the girder of Fig. 5*a* and assume that it carries one-half a railway track on the top flange. One engine wheel weighing 50,000 lbs., including impact, will then be carried in a space of about

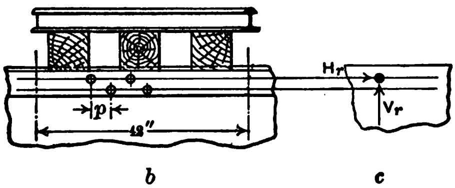


FIG. 5.

42″, it being the custom to asssume that three ties carry one axle load. Let H_r and V_r of Fig. 5*c* represent the resultant horizontal and vertical forces on the rivet as applied to the rivet by the girder web; the resultant of these two forces R should not exceed the ability of the rivet to safely resist either shear or crushing. To determine the pitch for this case,

Let W = the wheel load including impact (or other concentrated load);

a = total flange area

a_1 = total flange area with the part of web used for flange deducted;

s = three times the distance c. to c. of ties in inches,

h = effective depth of the girder in inches;

h_r = horizontal increment of stress in a length of one inch carried by a_1;

v_r = vertical load per inch of length of flange affected by the concentrated load,

r_r = resultant of h_r and v_r, $=\sqrt{h^2_r + v^2_r}$

$$=\sqrt{\left(\frac{V}{h}\cdot\frac{a_1}{a}\right)^2+\left(\frac{W}{s}\right)^2}.$$

Then if $R=$ the value of the rivet, as before,

$$p=\frac{R}{r_r}=\frac{R}{\sqrt{\left(\frac{V\cdot a_1}{h\cdot a}\right)^2+\left(\frac{W}{s}\right)^2}}.$$

Let this be applied to finding the pitch for connecting the top flange angles to the web for the end of the girder of Fig. 5*a*.

$$\frac{Va_1}{ha}=\frac{193{,}500\times 9.52}{57.24\times 12.33}=\frac{193{,}500\times 9.52}{705.77}=2610;$$

$$\frac{W}{s}=\frac{50{,}000}{42}=1190;$$

$$p=\frac{7876}{\sqrt{2610^2+1190^2}}=\frac{7876}{2710}=2.91''.$$

For the heavier concentrated loads, such as columns bring when carried on the flanges of the girders, the flange angles will often not contain enough rivets for security, and stiffeners must be used, as will be explained later. The foregoing gives a simple method for finding the maximum permissible pitch of the rivets connecting the vertical legs of the flange angles to the web.

The determination of the pitch of the rivets for con-

necting the cover plates to the horizontal legs of the flange angles is not a simple matter if theoretical accuracy is desired.

At *b*, Fig. 5*a*, the point where the first cover plate begins the flange angles have in them all the stress they can carry. It is clear, therefore, that from *b* to *c* with the rivets connecting the first cover to the flange angles spaced to take only the flange stress increment, the flange angles simply transmit the increments of stress to the first cover plate. At *c* the first cover plate and angles have all the stress they can carry, and with rivets spaced as before the increments of flange stress from *c* to the point of maximum flange stress are simply transferred through the angles and first cover to the second cover. Rivets connecting a cover plate to a flange are generally spaced to take the increments of flange stress which occur from the end to the point of maximum stress in the cover. If n be the number of lines of rivets connecting the cover to the flange then

$$p=\frac{nRh}{V}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5')$$

Assuming two lines of rivets in the cover plates the pitch at *b* for connecting the first cover to the flange angles $=\frac{2\times 7216\times 57.24}{153,000}=5.41''$. This is the maximum that may be used; the actual pitch would be made considerably less and usually a multiple of $\frac{1}{2}$, $\frac{3}{4}$, or $1''$ and at the same time such a pitch as would stagger well with the rivets in the vertical legs of the flange angles. If the maximum

pitch permissible were used the stress per square inch in the first cover plate would be zero at b and increase to 16,000 at c. This is highly undesirable, as there would exist in the girder in juxtaposition the angles with a unit stress of 16,000 and a cover with an average of 8000 lbs. The girder flange material cannot act in any such way without undue bending stresses on the rivets.

For this important reason cover plates should be made longer than a mere consideration of their relation to the moment polygon would require. The additional length, designated by y in formula (3) required, is a function of the number of rivets necessary to equalize the flange unit stress in all the flange material, and may be determined as follows:

Let n = number of rows of rivets in the cover plates;

a = area of flange without the cover plate under consideration, with unit stress s;

a_1 = area of flange, including the cover plate under consideration, with unit stress s_1;

p = pitch in inches of the cover plate rivets in each line;

R = value of one rivet connecting the cover to the flange;

y_1 = additional length of cover plate required at each end in ft. $=\frac{y}{2}$. $y_1=\frac{(a_1-a)s_1}{12Rn}\cdot p$.

$$y=\frac{(a_1-a)s_1}{6R\cdot n}\cdot p=\frac{S\cdot a\cdot p}{6a_1\cdot R\cdot n}(a_1-a). \quad . \quad . \quad . \quad . \quad (6)$$

The value of y for the first cover of the first example used to illustrate the method of finding the lengths is:

$$y=\frac{16{,}000\times 12.33\times 6}{6\times 18.33\times 7216}\cdot\frac{p}{n}=\frac{3}{2}\cdot\frac{p}{n}.$$

If the rivets be placed 3″ apart in two rows $y=2.25$.

If enough rivets are used at the end of a cover plate as at b of Fig. 5a to transmit to it its full proportion of the flange stress, the rivet pitch required to connect the cover plates to the flange at any point will be the following:

$$p=\frac{A_1}{A}\cdot\frac{n\cdot Rh}{V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5'')$$

In which A_1 is the total flange area, and A the area of all the cover plates, at the point under consideration. The number of rivets as determined by (5′) and (5″) should be increased by 20% to allow for the bending stresses. To still further limit these bending stresses no rivet passing through angles and covers should have a grip more than five times the diameter of the rivet.

PROBLEMS

5*a*. Using unit stresses of 10,000 and 20,000 lbs. for rivets in shear and crushing, compute the required rivet pitch for connecting the bottom flange angles to the web at c of Fig. 5a.

5*b*. For the data of Prob. 5a compute the rivet pitch for connecting the top flange angle to the web at a point vertically over c, including the effect of a locomotive wheel load.

5*c*. Using the unit stresses of 5a compute the value of y for the second cover plate.

5*d*. Compute the pitch of the rivets in the cover plate of 5c just adjacent to the portion y.

ART. 6. WEB PLATES

The maximum stresses that the web of a beam or girder must resist are of varying intensity and direction throughout its area.

For any elementary parallelopiped cut from the body of a beam in equilibrium subject to flexure, the forces on the faces of the element perpendicular to the plane of the drawing may be represented for the most general case

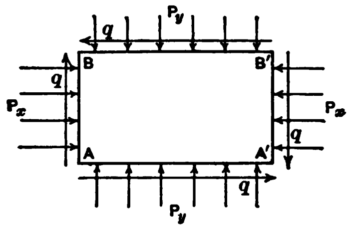


FIG. 6a.

as shown. The weight of the elementary particle may be neglected, as it is an infinitesimal of the third order as compared with the amount of stress on the faces, which are infinitesimals of the second order.

As the length of the sides of the particle approach zero as a limit, the intensities of the oppositely directed forces p_y are equal, and also the forces p_x. If the oppositely directed normal forces are of equal intensity the tangential or shearing forces q are also of equal intensity. The intensity of the shearing stresses at any point in a homogeneous prismatic body subject to flexure is given

from $q=\frac{VQ}{bI}$, a well-known formula of Mechanics of Materials, in which

q = the unit shearing stress;
V = the total shear;
b = the width of the body at the point under consideration;
Q = statical moment of the portion of the body above the point under consideration with reference to the neutral axis of the body, and
I = moment of inertia of the body with reference to the neutral axis.

The intensity of stress p_x at any point is given from the flexure formula $p_x=\frac{My}{I}$, in which the quantities are too well known to need definition.

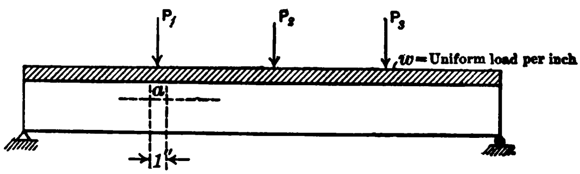


Fig. 6b.

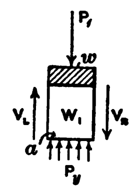


Fig. 6c.

The intensity of stress p_y may be found as follows: Let Fig. 6b represent any beam subject to flexure and a any point in the beam. Assume two planes distant 1″ apart on either side of a as shown by the dotted lines.

The vertical forces on the portion of the beam above a and between the parallel lines are as indicated on the larger scale sketch, Fig. 6c, in which V_R is the sum of

the shearing resistances of the right surface above a, V_L is the sum of the shearing resistances of the left surface above a.

$W=(P_1+w+W_1)=$the total load between the two assumed planes above the point a, and $A=$the area in square inches of the horizontal surface through a. Then

$$V_L-V_R-W+p_yA=0,$$

or

$$p_y=\frac{V_R+W-V_L}{A}. \qquad (7)$$

Care must be taken in giving the proper signs to V_R and V_L.

Having a method for finding p_x, p_y and q for any point in a beam, there remains to be found the plane in which their combined action has the greatest intensity.

The stresses per square inch p_x, p_y and q may be produced in any manner, even in an arbitrary manner or p_x and q, and p_y and q may be the rectangular components into which the actual stresses on any elementary parallelopiped have been resolved, as far as the following argument is concerned.

Let Fig. 6*d* be a redrawing of 6*a*. Let the plane OP be passed through the elementary particle perpendicular to the face and making the angle θ with AA', consider the elementary figure to have a dimension of unity perpendicular to the plane of the drawing.

Let it be required to find θ so that the stresses on OP will be normal and therefore either the maximum or minimum stresses in the body.

Let p represent the intensity of stress on OP, then

$$\overline{OP}\times p\times\frac{AP}{OP}=p_x\times AP+q\times AO,\quad \text{or}\quad p-p_x=q\cot\theta. \qquad (a)$$

and

$$\overline{OP}\times p\times\frac{AO}{OP}=p_y\times AO+q\times AP,\quad \text{or}\quad p-p_y=q\tan\theta. \qquad (b)$$

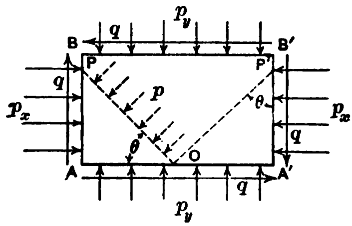


FIG. 6d.

Subtracting the second form of (b) from (a),

$$p_y-p_x=q(\cot\theta-\tan\theta)=q\cdot\frac{1-\tan^2\theta}{\tan\theta}=q\cdot\frac{2}{\tan 2\theta}, \qquad (8)$$

or $\tan 2\theta=\dfrac{2q}{p_y-p_x}$. The solution of this gives two values of θ, one of which corresponds to OP and the other to OP'.

Further, multiplying (a) by (b),

$$(p-p_x)(p-p_y)=q^2\tan\theta\cot\theta=q^2$$

$$p^2-p(p_x+p_y)=q^2-p_xp_y$$

$$p^2-p(p_x+p_y)+\left(\frac{p_x+p_y}{2}\right)^2=\left(\frac{p_x+p_y}{2}\right)^2+q^2-p_xp_y$$

$$p-\frac{p_x+p_y}{2}=\pm\tfrac{1}{2}\sqrt{(p_x+p_y)^2+4(q^2-p_xp_y)}$$

$$p=\tfrac{1}{2}[p_x+p_y\pm\sqrt{(p_x+p_y)^2+4(q^2+p_xp_y)}]$$

$$=\tfrac{1}{2}[p_x+p_y\pm\sqrt{(p_x-p_y)^2+4q^2}]. \qquad (9)$$

The two values of p given from (9) correspond to the maximum and minimum values and occur on either OP or OP'.

For an extended treatment of the subject of combined stresses see Wood's Analytical Mechanics, Chapter V.

If either or all of the quantities p_x, p_y or q have a direction opposite to that shown in Fig. 6d they should be entered in the formulas with a minus sign.

The planes of maximum shear make angles of 45° with the lines of maximum direct stress.

The amount of p_y is small, except for large concentrated loads, with reference to p_x and q. If p_y be neglected, at the end of the beam where $p_x=0$ the angle $\theta=45°$ and $p=q$, that is the direct stresses are equal to the shears in intensity and make an angle of 45° with them.

If p_y be neglected, at all points on the neutral axis where p_x is zero, $p=q$ and makes an angle of 45° with the axis.

In determining the location of the plane on which p, the principal stress intensity, is a maximum note, the values of the component stresses and inspection will show about the direction the resultant must have. If difficulty occurs in applying formula (9) it will be better to derive a special formula for the case at hand.

PROBLEM

6*a*. A plate girder of 14′ effective length carries a total load of 10,000 lbs. per linear foot of length, the load being applied to the top flange. Consider the structure to be solid and of a cross section composed of 4 angles $4'' \times 4'' \times \frac{1}{2}''$ and 1 plate $24 \times \frac{1}{2}''$, as shown in the sketch, determine the maximum stress intensity and the direction of

its action at each numbered point as is indicated for points Nos. 3, 7, 9, 11, and 15 of Fig. 6e.

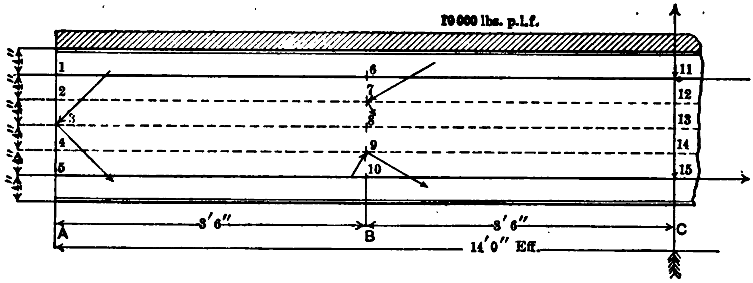


Fig. 6e.

This problem is so important for a proper understanding of the stress distribution on web plates that it is completely solved for points Nos. 3, 7, 9, 11 and 15, and much of the computation made for the remaining points, in what follows in this article. In Fig. 6e arrows acting toward a point indicate compression and away from the point tension.

The computations for p_x, p_y and q should be made about as follows:

$$
\begin{aligned}
I_x = \tfrac{1}{12} \times \tfrac{1}{2} \times (24)^3 &= 576.00 \\
5.56 \times 4 &= 22.24 \\
4 \times 3.75 \times 10.82^2 &= 1756.08 \\
&= 2354.32 \\
M_A &= 0 \\
M_B &= 2{,}205{,}000 \text{ in.-lbs.} \\
M_C &= 2{,}940{,}000 \text{ in.-lbs.}
\end{aligned}
$$

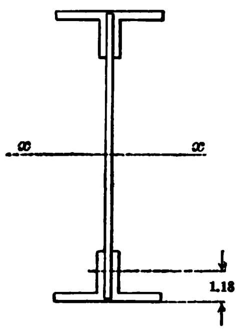


Fig. 6f.

At points Nos. 1, 2, 3, 4 and 5, $p_x = 0$

At points Nos. 7 and 9, $p_x = \dfrac{2,205,000 \times 4}{2354.32} = 3,740$

At points Nos. 11 and 15, $p_x = \dfrac{2,940,000 \times 8}{2354.32} = 10,000$

$$\begin{array}{rl} Q_{11} = 2 \times 3.75 \times 10.82 = & 81.15 \\ +2.00 \times 10.00 = & 20.00 \\ \hline & 101.15 \end{array}$$

$$\begin{array}{rl} Q_7 = 2 \times 3.75 \times 10.82 = & 81.15 \\ 2.00 \times 10 \quad = & 20.00 \\ 2.00 \times 6 \quad = & 12 \\ \hline & 113.15 \end{array}$$

$$Q_3 = 113.15 + 2 \times 2 \quad = 117.15$$

$q = V \cdot \dfrac{Q}{bI} =$ for

$$\text{No. } 3 = V \cdot \frac{117.15}{.5 \times 2354.32} = 70,000 \times .0993 = 6960$$

$$\text{No. } 7 = V \cdot \frac{113.15}{.5 \times 2354.32} = 35,000 \times .0960 = 3360$$

$$\text{No. } 11 = V \cdot \frac{101.15}{.5 \times 2354.32} = \quad 0000 \times .0853 = 0000$$

From formula (7) there is found

at No. 3, $p_y = \dfrac{833\frac{1}{3} - 833\frac{1}{3} \times .5}{.5} \qquad = 830$

at No. 7, $p_y = \dfrac{833\frac{1}{3} - (833\frac{1}{3}).304}{.5} \qquad = 1240$

at No. 11, $p_y = \dfrac{833\frac{1}{3} - (416\frac{2}{3} + 416\frac{2}{3}).121}{.5} = \dfrac{733}{.5} = 1470$

Values of Q, $\dfrac{Q}{bI}$, and $\dfrac{VQ}{bI}$ should be tabulated as follows:

	Q	$V \times \frac{Q}{bI}$
12″	= 00.000	V .0000
11″	= 58.375	V .0165
10	= 74.125	V .0210
9	= 88.375	V .0250
8	= 101.125	V .0853
7	= 104.875	V .0883
6	= 108.125	V .0919
5	= 110.875	V .0936
4	= 113.125	V .0960
3	= 114.875	V .0976
2	= 116.125	V .0986
1	= 116.875	V .0992
0	= 117.125	V .0993

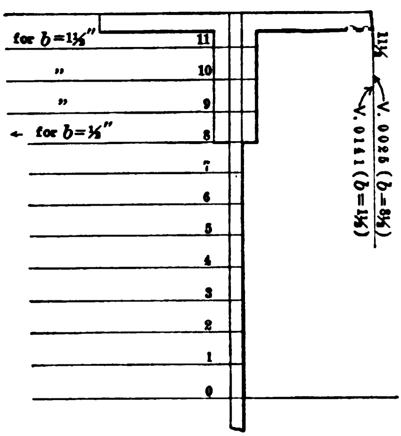


FIG. 6g.

Let A_1 = the area above division No. 11 = 4.25 + .75 = 5.00 sq.ins.

A_2 = the area above division No. 10 = 5.00 + 1.50 = 6.50 sq. ins., etc.

Values of $q \times$ areas:

$q \times A_1 = V \times .017$	$q \times A_{13} = V \times .550$
$A_2 = V \times .046$	$A_{14} = V \times .599$
$A_3 = V \times .081$	$A_{15} = V \times .648$
$A_4 = V \times .121$	$A_{16} = V \times .696$
$A_5 = V \times .165$	$A_{17} = V \times .743$
$A_6 = V \times .210$	$A_{18} = V \times .790$
$A_7 = V \times .257$	$A_{19} = V \times .835$
$A_8 = V \times .304$	$A_{20} = V \times .879$
$A_9 = V \times .352$	$A_{21} = V \times .919$
$A_{10} = V \times .401$	$A_{22} = V \times .954$
$A_{11} = V \times .450$	$A_{23} = V \times .983$
$A_{12} = V \times .500$	$A_{24} = V \times .1000$

At No. 3:

$$\tan 2\theta = \frac{2 \times 6960}{830 - 0} = \frac{13{,}920}{830} = 16.771$$

$$2\theta = 86^\circ - 36''$$

$$\theta = 43 \quad - 18''$$

$$p = \tfrac{1}{2}[0 + 830 \pm \sqrt{(830)^2 + 4(6960)^2}]$$

$$= \tfrac{1}{2}[830 \pm 13{,}940]$$

$$= +7380 \quad -6550$$

At No. 7:

$$\tan 2\theta = \frac{2 \times 3360}{1240 - 3740} = \frac{6720}{2500} = 2.683$$

$$2\theta = 68^\circ - 10'$$

$$\theta = 34^\circ - 05'$$

$$p = \tfrac{1}{2}[3740 + 1240 \pm \sqrt{(2500)^2 + 4(3360)^2}]$$

$$= \tfrac{1}{2}[4980 \pm 7170]$$

$$= +6080, \ -1100.$$

At No. 9:

$$p_x = -3740$$

$$p_y = \frac{833\tfrac{1}{3} - 833\tfrac{1}{3} \times .696}{.5} = 510$$

$$q = 3360$$

$$\tan 2\theta = \frac{2 \times 3360}{510 + 3740} = \frac{6720}{4260} = 1.59$$

$$2\theta = 57^\circ - 50'$$

$$\theta = 28^\circ - 55'$$

$$p = \tfrac{1}{2}[-3740 + 510 \pm \sqrt{(4250)^2 + 4 \times \overline{3360}^2}]$$

$$= \tfrac{1}{2}[-3230 \pm 7950]$$

$$= -5590, \ +2360.$$

At No. 11:

$$\tan 2\theta = \frac{2 \times 0}{1470 - 10{,}000} = \text{an infinitesimal}$$

$$\theta = 0$$

$$p = \tfrac{1}{2}[10{,}000 + 1470 \pm \sqrt{(8530)^2 + 4 \times 0}]$$
$$= \tfrac{1}{2}[11{,}470 \pm 8530]$$
$$= +10{,}000, +1470.$$

At No. 15:

$$p_x = -10{,}000$$

$$p_y = \frac{833\tfrac{1}{3} - (416\tfrac{2}{3} + 416\tfrac{2}{3})}{.5} \cdot 879 = \frac{100}{.5} = 200$$

$$q = 0$$

$$\tan 2\theta = \frac{2 \times 0}{200 - 10{,}000} = \text{an infinitesimal} = \tfrac{1}{2}[-9800 \pm 10{,}200]$$
$$= -10{,}000, \quad +200$$

$$0 = 90 \text{ or } 0$$

$$p = \tfrac{1}{2}[-10{,}000 + 200 \pm \sqrt{(10{,}200)^2 + 4 \times (0)^2}]$$

ART. 7. AREA OF WEB PLATES

The preceding article has shown that the stresses in the web of a plate girder are of greatly varying directions and intensities. The tensile stresses need an adequate amount of material to resist them, the amount of which is a direct function of the stress.

The action of the compressive stresses is augmented to a much greater degree than for tension, by secondary stresses if the material be even very slightly lacking in uniformity. That is, a certain kind of column action is set up.

The maximum tensile and compressive stresses in the

web cross each other at right angles; at the neutral axis they are approximately equal, and make angles of approximately 45° with the axis and equal the shearing stresses in intensity; above the neutral axis the compressive stress intensity is increasing while the tensile is decreasing; below the axis the opposite relation exists.

The shearing stresses are of the greatest intensity at the neutral axis, on horizontal and vertical planes, for any cross-section. At the cross-section where the greatest resultant shear occurs, there, on the neutral axis will be found the maximum shearing unit stresses. For simple spans the shear is a maximum at the ends of the span.

It is the practice to design web plates so that they will resist the shear at the point of maximum shear, and have enough thickness to provide ample bearing for the rivets connecting flange angles to the web; this latter very frequently determines the thickness of the web plate. The design of the web plate is also governed by practical considerations, such as requirements for handling in shop and field and requirements to resist corrosion.

The best practice requires as a minimum for the thickness of girder webs: $\frac{7}{16}''$ for railway, $\frac{3}{8}''$ for highway, and $\frac{5}{16}''$ for building work. It is the practically universal custom to make the web plate, for a girder, of uniform section throughout its length.

The shearing unit stress should be about 20% less than that used for direct tension, that is, where 16,000 is used for tension, 13,000 is a consistent unit for the maximum shearing stress intensity. Where the shearing stress is assumed to be uniformly distributed over the girder

web this unit should be reduced to 10,000 lbs. per square inch and applied to the net section.

The web plate is also designed to resist any tendency to buckling or sidewise failure. The tensile stresses which act on any element of the web tend to pull the element back into its proper position if from any cause the element is displaced laterally. In the portion of the web below the neutral axis the excess of tensile over compressive stress would give a component perpendicular to the web which would tend to take out any buckle that might exist. In the portion above the neutral axis the corresponding component would tend to increase any buckle. This can readily be seen by a study of Fig. 7*a*.

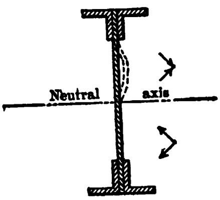


Fig. 7*a*.

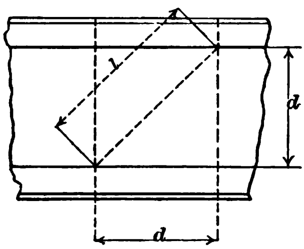


Fig. 7*b*.

In order to determine whether a web needs stiffening an investigation which will lead to a safe structure can be made on the following basis:

Formulas (8) and (9), of Art. 6, show that at the neutral axis of the girder the maximum resultant compressive stress intensity approximately equals the maximum shearing stress intensity and that the line of its action makes an angle of about 45° with the neutral axis. A compressive unit stress equal to the maximum shearing

stress intensity is therefore assumed to act along the element of the web having a length of l in Fig. 7*b*, and this must be less than the permissible unit stress for a column of this length. Assuming that a safe unit stress for a column is given from the formula

$$P = 16{,}000 - 70\frac{l}{r},$$

the notation for which needs no definition, the 16,000 should be reduced to 12,000 if the average shearing unit stress is used, as the maximum unit compression. The formula then becomes

$$P = 12{,}000 - 52.5\frac{l}{r}.$$

Substituting for l and r their values,

$$l = d\sqrt{2}, \quad \text{and} \quad r = .29t,$$

where t = the thickness of the web.

The formula then is

$$P = 12{,}000 - 52.5\frac{d\sqrt{2}}{.29t} = 12{,}000 - 257.5\frac{d}{t} \quad . \quad . \quad (a)$$

As the tension in the web below the neutral axis will prevent buckling, the value of d should be $\frac{d}{2}$, and as the tension in the upper half of the web helps to prevent buckling the constant may be divided again by 2, the formula then being

$$P = 12{,}000 - 65\frac{d}{t}. \quad . \quad . \quad . \quad . \quad . \quad (10)$$

It should be noted that d equals either the clear distance between flange angles or stiffeners.

To show the method of applying this formula to see if web stiffeners are necessary let the girder of Fig. 5*a* be examined.

The average shearing unit stress $=\dfrac{193{,}500}{22.5}=8580$ lbs. per square inch.

The allowable unit stress $=12{,}000-\dfrac{65\times 48.5}{.375}=12{,}000$ $-8420=3580$ lbs. per square inch.

As the actual stress exceeds the allowable, the web must be made thicker or the stiffeners placed closer together or else both combined.

If the web be made $\frac{1}{2}''$ thick

The average shearing unit stress $=\dfrac{193{,}500}{30}=6450$ lbs. per square inch.

The allowable shearing unit stress $=12{,}000-\dfrac{65-48.5}{.5}$ $=12{,}000-6300=5700$ lbs. per square inch, which shows that the web must be further thickened or the stiffeners placed closer together.

If the web be made $\frac{7}{16}''$ thick and stiffeners placed 30″ apart in the clear the average shearing unit stress $=\dfrac{193{,}500}{26.25}=7380$ lbs. per square inch.

The allowable shearing unit stress $=12{,}000-\dfrac{65\times 30}{.44}$ $=12{,}000-4420=7580$ lbs. per sq.in.

PROBLEMS

7a. The accompanying sketch shows the cross-section of the girder of Prob. 6a and the curve of unit shear distribution. Construct the

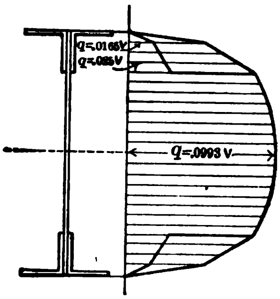


FIG. 7c.

figure to three times the size of the sketch shown and write in the value of the ordinates to the curve at points 1″ apart from the top to the bottom of the section.

7b. Are stiffeners required on the web of the girder of Prob. 6a? If not, show it by computation.

7c. What is the minimum thickness permissible for the web of the girder in Prob. 6a?

ART. 8. STIFFENERS FOR WEB PLATES

Stiffeners have several important functions, the chief of which are:

(a) They keep the web from buckling, due to the compressive stresses in it.

(b) They help hold the compression flange from lateral failure as a whole and from failure in detail in any direction.

(c) They are used to relieve the rivets connecting the

loaded flange to the web, by transferring the load directly to the web.

(*d*) They are used to reduce to proper amount the vertical stresses, on horizontal planes, in the web brought by local concentrated loads.

(*e*) They help hold the web true to shape during manufacture and erection.

According to their principal functions they should be divided into two general classes, *intermediate stiffeners,* which are chiefly used for the purposes of (*a*), (*b*) and (*e*); *concentrated load stiffeners,* for (*c*) and (*d*), although each class performs all the functions of course. Intermediate stiffeners act as a beam standing vertically to resist lateral displacement. They are generally made of angles in pairs and riveted to the girder web as shown in Fig. 8*a*. The

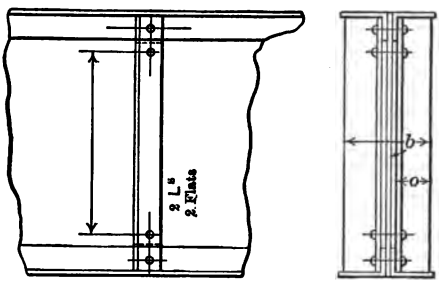


FIG. 8*a*.

legs of the stiffening angles which are against the fillers are near the neutral axis of the pair and therefore should be only large enough to receive the connecting rivets; a $3\frac{1}{2}''$ leg for $\frac{7}{8}''$ rivet and $3''$ leg for $\frac{3}{4}''$ are enough.

The width *b* of the outstanding legs is the principal

element of efficiency for resistance to transverse displacement. A common rule is to make the width of the outstanding leg (O) equal to the depth of the girder (d) in inches divided by 30 plus 2″, or expressed as a formula

$$O=\frac{d}{30}+2''. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The following is a common method of locating intermediate stiffeners.

Intermediate stiffeners should be used at points as required by (10) or wherever the unsupported depth between flange angles is more than 160 times the web thickness. Where intermediate stiffeners are required they should never be further apart in the clear than the clear distance between the flange angles, with a maximum limit of 5′.

Concentrated load stiffeners act to transfer a concentrated load into the web or to transfer the girder load to a support. The sketch of Fig. 8*b* will illustrate both cases.

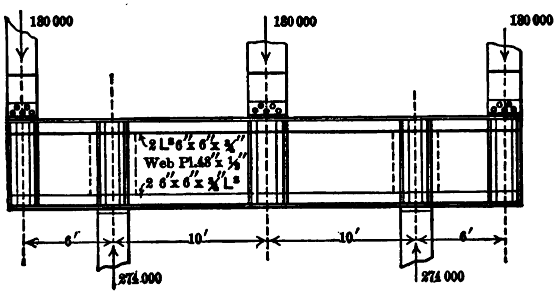


Fig. 8*b*.

The girder here shown being loaded with three concentrated loads of 180,000, and its own weight of 250 lbs. per linear foot. The lines of maximum web stress

for a girder of this character are very different from those of the girder of problem 6*a*. If such a girder were constructed without stiffeners, and formula (7) were applied to finding the intensity of p_v, this intensity would be found to be very high. The heavy vertical stiffeners are added to transfer the point of application of these loads from the outer edges of the flanges to an average position of the center of the web, and thereby decrease the intensity of vertical stress on horizontal planes just at the inner edge of the flange angles. These stiffeners cannot fully accomplish their object, as they are of elastic material and they are shortened by compression, and, as they are fastened to the web, the web and stiffeners move together except for a variation due to the deformation of the rivets. The stiffeners also serve to relieve the flange rivets from the component due to the vertical load.

Stiffeners supporting concentrated loads may be designed as columns with the formula $P = 16{,}000 - 70\frac{l}{r}$, in which l should be taken as one-half the girder depth and r as the radius of gyration about an axis in the longitudinal center line of the girder.

Wherever the combined stress intensity on a web plate exceeds the allowable unit stress the stresses may be reduced by means of side plates, as shown by dotted lines.

Where the concentrated load stiffeners are at the end of a girder they should have their outstanding leg about equal to the horizontal leg of the flange for the sake of good appearance, and where it is desired to face the ends of such girders the stiffening angles must be made thicker

by $\frac{1}{8}''$ than theoretical considerations determine, to allow for material removed by such facing.

PROBLEMS

8a. Determine the size of the end stiffeners for the girder of Fig. 5*a*, using two pairs of angles, one on the outer and one on the inner edge of the sole plate.

8b. Assuming the wall plate to be 18″ long for Fig. 5*a*, determine the size and spacing of the intermediate stiffeners for the girder.

8c. Determine the sizes of the stiffeners at the points of concentrated loading for the girder of Fig. 8*b*.

8d. Determine the sizes and spacing of the intermediate stiffeners for the girder of Fig. 8*b*.

ART. 9. SPLICES FOR WEB PLATES

Webs of plate girders should not be spliced unless necessary, but as wide plates cannot be procured in one piece, in general, for girders over 60 ft. long, splicing for long girders is unavoidable. The manufacturers furnish lists of the maximum dimensions of their plates. The number and location of the web splices should be determined by a combined consideration of the cost and resulting efficiency of the possible arrangements. The splice should be designed with the idea of transferring the stresses in the web across the cut in the most direct manner possible, just as in plate girder design as a whole the effort should be made to have the stresses developed as indicated by the theory of flexure for solid beams. Let it be required to splice the web at the center of the girder which has an effective length of 80′, and a composition as shown in Fig. 9*a*.

The girder has been designed on the assumption that

$\frac{1}{8}$ the gross area of the web is available as flange area. The maximum bending moment $= 6{,}300{,}000$ ft.-lbs. The center shear simultaneous with this moment $= 0$. The maximum center shear $= 75{,}000$ lbs., and the bending moment simultaneous with this $= 3{,}400{,}000$ ft.-lbs.

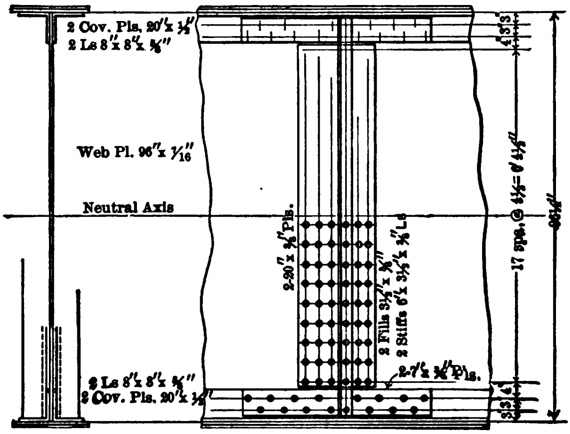


FIG. 9a.

The flange area is made up of:

$$\tfrac{1}{8} \text{ web} = 96 \times \tfrac{7}{16} \times \tfrac{1}{8} = 5.25 \text{ sq.in.}$$

$$2 \text{ angles } 8 \times 8 \times 32.7 = 19.23 - 2.50 = 16.73$$

$$1 \text{ cover plate } 20 \times \tfrac{1}{2} = 10 - 1.00 = 9.00$$

$$1 \text{ cover plate } 20 \times \tfrac{1}{2} = 10 - 1.00 = 9.00$$

a total of 39.98 sq.ins.

The simplest form of web splice consists of two vertical plates terminated at the top and bottom by the vertical legs of the flange angles, but this makes a normal distribution of the web stresses impossible in the vicinity of the splice. The resultant of all the forces on any beam cross-section may be represented by a shear and a couple

as is well known from mechanics, but the proper form of splice should be designed with reference to the distribution of these forces. The portion of the web between the flange angles is so near the extreme fiber that the stresses may be assumed to be horizontal throughout. The flange unit stress has been taken at 16,000 lbs.; the bearing and shearing values for rivets will be taken to correspond to this at 24,000 and 12,000 lbs. per square inch respectively, or at 50% more and 25% less than for tension. The thickness of the splice plates should be sufficient to transmit the maximum possible stresses which can occur at the point of maximum stress in the splice.

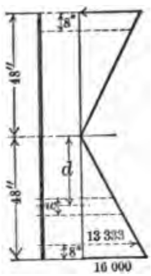


FIG. 9*b*.

The total stress on the portion of the web between the flange angles $= (8'' - 1'') \times \frac{7}{16} \times 14{,}667 = 44{,}800$ lbs.; the least that can be used for this part of the splice will be $2-7 \times \frac{3}{8}$ flats for each flange. The net area of these two plates $= 4.5$ sq.in., while that of the strip of the web $= 3.06$ sq.in. The bearing value of the rivets is less than that for double shear, hence one rivet is worth 9190 lbs. at the edge of the web plate and $9190 \times \frac{43.75}{48} = 8350$, an average for this part of the splice. The number of rivets on each side of the splice for splicing the strip $= \frac{44{,}800}{8350} = 5.37$ or 6.

For the condition of maximum moment there is no shear at the splice and the web stresses are horizontal.

The number of vertical rows of rivets is evidently a

function of the vertical pitch, the more vertical rows the greater the vertical pitch.

Let w = the pitch of the rivets in a vertical direction;

n = number of vertical rows of rivets;

d = distance from the neutral axis to horizontal row of rivets.

For three vertical rows

$$(w-1)\tfrac{7}{16}\times 16{,}000\times d/48=3\times 9190\times d/48$$

or

$$w=34.570/7000=4.94'', \text{ say } 4\tfrac{1}{2}''.$$

For two vertical rows

$$(w-1)\tfrac{7}{16}\times 16{,}000\times d/48=2\times 9190\times d/48$$

or

$$w=25{,}380/7000=3.62, \text{ say } 3''.$$

For this splice 3 vertical rows on each side spaced $4\frac{1}{2}$ will be used, as it gives a web plate with less reduction of strength at the splice.

The splice should now be examined for the case of 75,000 shear and simultaneous moment of 3,400,000 ft.-lbs.

From (5) the flange pitch at the center $p=\dfrac{9190\times 96}{75{,}000}$ $=11.6''$ for no part of the web acting as flange. The flange as designed uses 5.25 sq.ins. of the web for flange area; the pitch of the rivets connecting the flange angles to the web is $\dfrac{9190\times 96}{75{,}000\times\dfrac{34.73}{39.98}}=11.6\times\dfrac{39.98}{34.73}=13.3''.$

The $7\times\frac{3}{8}$ flats on the vertical legs of the flange angles will therefore be made to take about 6 or 8 rivets on each side of the splice.

For a web splice at some point other than the center of the girder, the number of rivets in the flange plates on the side toward the center will be the number required for splicing the web; on the side toward the end the number will be the sum of those for connecting flange to web plus those for web splice.

The extreme fiber stress on the web for a moment of 6,300,000 ft.-lbs. was 16,000; for a moment of 3,400,000 ft.-lbs. it is 8630. For the strip of web between the flange angles the previous determination is evidently ample. A study of the stress intensity at different points in the web for this condition of loading would show that the stress intensity was nowhere as great as for the case for which the splice was designed.

A safe resultant rivet bearing of 9190 should not be exceeded.

The vertical component on a rivet at either edge of the main splice plates will not exceed $75,000/54 = 1390$ lbs., the horizontal component $= 3.5 \times \frac{7}{16} \times 8630 \times 38/48 \times \frac{1}{3} = 3480$ per rivet. The resultant $= \sqrt{\overline{1390}^2 + \overline{3480}^2} = 3800.$

PROBLEMS

9a. What is the relative efficiency of the unriveted plate of Fig. 9*a* to that of the section of the plate through the first vertical line of rivets in the splice.

9b. Compute and compare the resisting moment of the rivets in the splice of Fig. 9*a* with that of the net section of the web plate through the first vertical line of rivets.

9c. Make a sketch of a splice for the girder, using only two vertical rows of rivets on each side of the splice.

ART. 10. SPLICES FOR THE COMPONENT PARTS OF THE FLANGES

Splicing any of the component parts of a girder flange should be avoided, as it is never necessary except to meet some emergency, or reduce the cost slightly. The flange angles and cover plates, for girders of the maximum length now used, may be obtained in one piece. The cost of a few very long pieces, however, may be so high, due to the expense of shipment in less than proper amounts, or the need of the completed structure may be so urgent, as to make it advisable to permit splicing of component parts of the flange.

Flange angle splices should be located when possible as follows:

a. Where there is an excess of flange section over the required amount;

b. Between adjacent pairs of stiffeners, where these pairs of stiffeners are far enough apart to permit a splice of proper length to be made;

c. So that not more than two flange angles shall be spliced in any cross-section of the girder;

d. So that the flange angles on opposite sides of the girder flange will be spliced on opposite sides of the center of the girder.

The splicing material should be disposed with reference to the shape of the section spliced. The splice for an angle with equal legs should be made by a cover angle with two equal legs. The net area of each leg of the splice

PROBLEMS

7a. The accompanying sketch shows the cross-section of the girder of Prob. 6*a* and the curve of unit shear distribution. Construct the

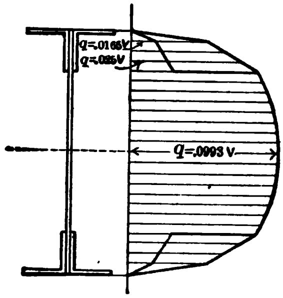


FIG. 7*c*.

figure to three times the size of the sketch shown and write in the value of the ordinates to the curve at points 1″ apart from the top to the bottom of the section.

7b. Are stiffeners required on the web of the girder of Prob. 6*a*? If not, show it by computation.

7c. What is the minimum thickness permissible for the web of the girder in Prob. 6*a*?

ART. 8. STIFFENERS FOR WEB PLATES

Stiffeners have several important functions, the chief of which are:

(*a*) They keep the web from buckling, due to the compressive stresses in it.

(*b*) They help hold the compression flange from lateral failure as a whole and from failure in detail in any direction.

(*c*) They are used to relieve the rivets connecting the

to act as flange area; for this girder, however, 2.81 sq.in. of the web was assumed to act as flange area. The value of I therefore is $2540 - 2540 \times \frac{2.81}{24.33} = 2540 - 290 = 2250$ lbs.

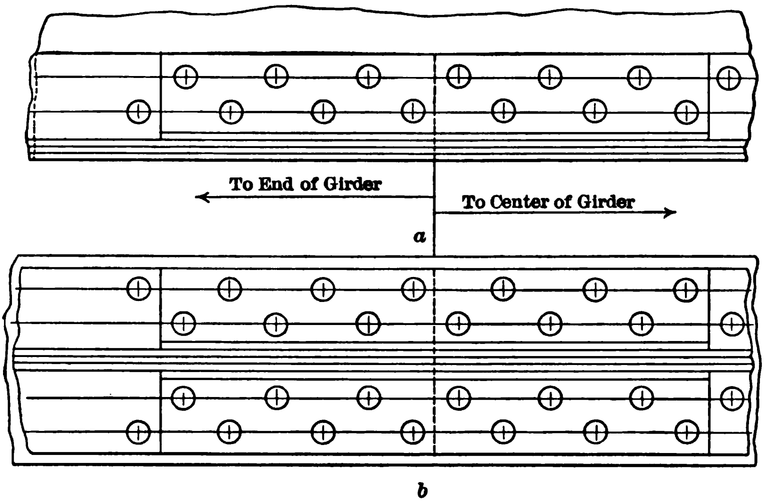


Fig. 10 (*a* and *b*).

Fig. 10 (*c*) shows the forces on a rivet through the web and vertical legs of the splice and flange angles due to increment I.

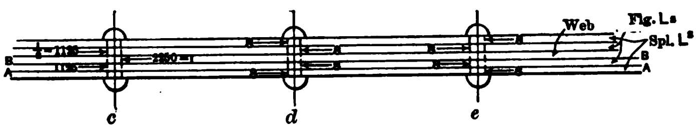


Fig. 10 (*c*, *d* and *e*).

Fig. 10 (*d*) shows the forces on a rivet through the web and vertical legs of the splice and flange angles due to splicing, for the left portion of the splice.

Fig. 10 (*e*) shows the forces on a rivet through the

web and vertical legs of the splice and flange angles due to splicing, for the right portion of the splice.

The unit stresses on the rivet will be taken at 12,000 lbs. per square inch for shear, and 24,000 lbs. per square inch for bearing.

Consideration of the forces on the rivets and the thickness of flange and splice material shows that the shearing value of the rivets is the determining one along the plane AA.

The number required in the vertical legs $=\frac{152,320}{4\times 7216}=5.3$ or 6.

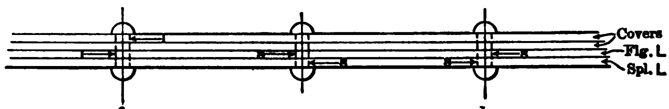


Fig. 10 (f, g and h).

Fig. 10 (f), (g) and (h) show the forces on the rivets through the horizontal legs of the flange angles of the top flange splice. It is readily seen that the number of rivets required $=\frac{152,320}{4\times 7216}=5.3$ or 6.

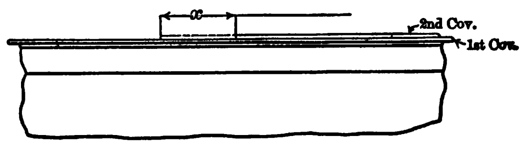


Fig. 10i.

Cover plate splices should be located so that a simple lengthening of an outer cover plate will form the splice. For example if a splice were required in the first cover

plate of the girder of Art. 4, it should be spliced by extending the second cover a distance x, which must be enough to permit the location of a proper number of rivets, shown by the dotted lines in Fig. 10 (i).

PROBLEM

10. Design a splice for the flange angles of the girder of Prob. 4a, which shall be located at the same point in the length of the girder as was the one used to illustrate this article.

ART. 11. CONNECTING ONE GIRDER TO ANOTHER

The problem of framing one girder into another is one so frequently met with that a connection for two adjacent stringers and the floorbeam between them will be designed to illustrate the method of making the computations for such connections.

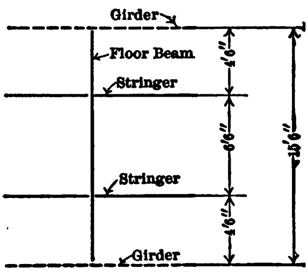


FIG. 11a.

Fig. 11a is a line drawing of a plan showing the relative position of girders, floorbeams, and stringers. A load on any stringer is carried first to the floorbeams at its ends. The floorbeam carries it to the girders and the girders to the abutments or other end supports.

The stringer is composed of 1 web plate $27 \times \frac{1}{2}$;
4 angles $5 \times 3\frac{1}{2} \times 13.6$ ($\frac{1}{2}''$).

The floorbeam is composed of 1 web plate $38 \times \frac{1}{2}$;
4 angles $5 \times 3\frac{1}{2} \times 19.8$ ($\frac{3}{4}''$)

The maximum stringer reaction is 97,800 lbs. ($ll = 48{,}800$ Imp. $= 46{,}600$ and $dl = 2400$).

The maximum reaction for two stringers is 131,700 lbs. ($ll = 66{,}300$, Imp. $= 60{,}600$ and $dl = 4800$).

The requirements of construction will be assumed to demand that the bottom flange of the stringer be $4\frac{1}{2}''$ above the bottom flange of the floorbeam. The connection between the stringer and floorbeam is made by two vertical angles which are riveted onto the stringer in the bridge shop. The rivets through the outstanding legs of the connection angles and the floorbeam web must be field driven. The unit stresses are 12,000 and 24,000 for shear and bearing for shop rivets and 20% less than these values for field-driven rivets; $\frac{7}{8}''$ rivets will be used and the connection angles will be made $\frac{1}{2}''$ thick so that after they are faced on the back of their outstanding legs they will not be less than $\frac{3}{8}''$ thick on these legs.

Fig. 11 (d) shows how the load W is applied to each rivet and carried to the stiffeners. The rivet should be examined for crushing by the web for W and for shear for $\frac{W}{2}$.

The number required for crushing $= \frac{97{,}800}{10{,}500} = 9.3$ or 10. For shear the number $= \frac{97{,}800}{2 \times 7216} = 6.8$ or 7. The upper and lower rivet in the lines xx and zz of (b) cannot be

counted for this connection, as they have other duty to perform. Therefore, in order that the rivets be not closer than 3″, a leg for the connection angles which will contain 2 rows of rivets must be used. As 10 rivets are required, a symmetrical arrangement demands 11. These rivets pass through fillers and a good requirement is to extend the fillers for such cases and put 50% additional rivets through

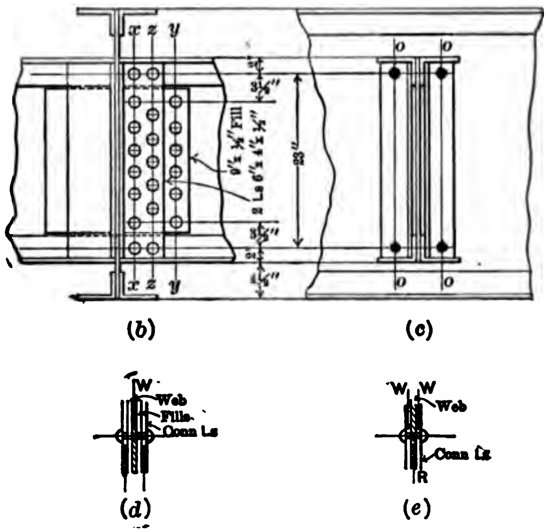


FIG. 11.

them. Many good designers consider it allowable to assume that fillers connected to web plates by a line of rivets such as *yy* increase the bearing value of rivets in lines *xx* and *zz* by the entire bearing value of the rivets in *yy*.

Consideration of (*c*) of Fig. 11 shows that it will be possible to get

16 rivets in the two lines *oo* and have them a little over 3″ apart, and

18 rivets in the two lines *oo* and have part of them $2\frac{3}{4}$ and part 3″ apart.

Fig. 11 (*e*) shows how the forces act on a rivet connecting the outstanding legs of the connection angles to the floorbeam web.

The bearing value of a field rivet in the $\frac{3}{8}$ (net thickness) stiffener leg is 6300.

The shearing value of a field rivet is 5770.

For the maximum stringer load the number of rivets required for shear $=\frac{97{,}800}{5770}=16.95$ or 18, which would enable two lines of rivets to be used, if part of the number are $2\frac{3}{4}''$ apart.

The rivets must also be examined for bearing on the floorbeam web when both stringers are bringing their combined maximum loads at this connection. The bearing value of a field rivet in the $\frac{1}{2}''$ floorbeam web $=8400$ lbs. The number of rivets required $=\frac{131{,}700}{8400}=15.6$ or 16, which is one less than the number which are required for maximum shear on the rivets for one stringer loaded.

The number of rivets by the second computation is usually, though not in this case, the larger and hence the governing one. The rivets should be located on the lines *oo* in (*c*) of Fig. 11. The two fillers under the connection angles and on the stringer web must be $\frac{1}{2}''$ thick and 3″ wider than the angle leg. They should therefore be $9''\times\frac{1}{2}''$ flats.

The requirement of two lines of rivets for the leg of the connection angle against the stringer web demands a

6″ leg. One line between each leg of the connection angle and the floorbeam web fixes the width of the outstanding leg at 3½ or 4″.

The connection angles cannot be less than ⅜″ in thickness. Examination of this net section through lines *xx* and *oo* shows them to be ample to resist shear. Sometimes they must be increased in thickness for this.

The connection angles are therefore made 6×4×16.2 (½).

PROBLEM

11. Design the connection between two adjacent stringers and their floorbeam. The stringer consisting of

1 web 24×$\frac{5}{16}$;

4 angles 5×3½×15.2 ($\frac{5}{16}$)

The floorbeam consisting of

1 web 34×½;

4 angles 5×3½×13.6 (½);

2 covers 11×½.

The shears being the same as for the example solved in this article.

ART. 12. END BEARINGS

The end bearings of a girder must receive the load brought to the end of the girder and distribute the same over the masonry or other support. For the single-track girder of Art. 4, which was taken to illustrate the method of finding lengths of cover plates an end reaction of 193,500 lbs. was assumed. For girders less in length than 60′ one end of the girder is bolted to the masonry and the other

allowed to slide on a plate. This arrangement makes the determination of the reactions possible.

The area of the bearing on the wall, assuming the bridge seats to be of granite and 600 lbs. per square inch as a unit stress, should be $\frac{193,500}{600}=323$ sq.in.

It is not advisable to make an end bearing of the simple nature indicated in Fig. 12 too long. The tendency of

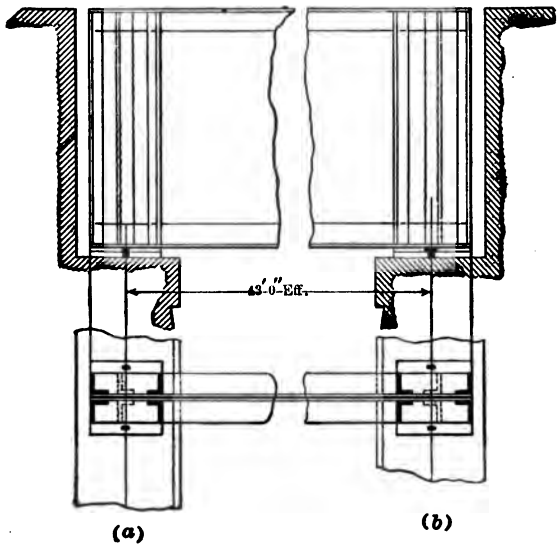


FIG. 12.

such a bearing is to overstress the bearing along its inner edge.

The upper plate of the two shown at each end is called the sole plate, and is connected by rivets, countersunk on the bottom, to the girder. The bottom of the sole plate at (*b*), the expansion end, should be planed.

The lower plate of the two shown at each end is called the wall plate.

The upper surface of the wall plate at (*b*), the expansion end, should be planed.

The wall plates are held in position by the two anchor bolts at each end.

The bearings at the ends of the girder, when of two simple plates, should not be too wide, as the tendency of the bearing is to overload the masonry along the portion covered by the bottom flange of the girder.

The bearing for this case will be made $18 \times 18''$.

The two plates which project beyond the bottom flange angles must be strong enough in flexure to distribute the load on the bridge seat. Each inch of length of the plates may be considered acted on in a transverse direction by the forces shown in (*d*).

The bending moment then

$=5400(9/2 - 6.18/2) = 5400 \times 1.41 = 7614$ in.-lbs.

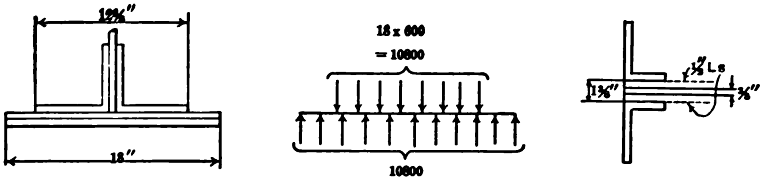
If the sole and wall plates be made of equal thickness, then the resisting moment of each plate must be 3807 in.-lbs.

The depth (or thickness) of plate required may be obtained from $M = \frac{SI}{c} = \frac{S \cdot bd^2}{6} = 3807$ or $d^2 = \frac{3807}{16{,}000} \times 6 = 1.43''$ and $d = 1.25''$ about.

These plates should neither of them be less than $\frac{3}{4}''$ thick, even if a less thickness would furnish proper strength.

For small spans generally only two pairs of stiffeners, one over the outer and the other on the inner edge of the bearing plates, are used.

The end stiffeners for this case should be $5 \times 3\frac{1}{2}$ angles and their radius of gyration in a direction transverse to the web is 2.78″ approximately. The required area for



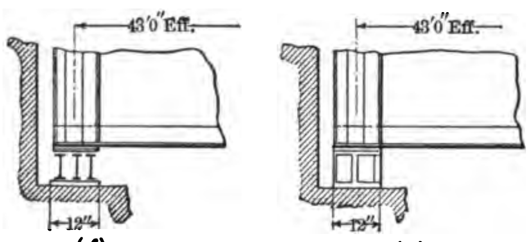
(c) (d) (e)

FIG. 12.

these stiffeners $= \dfrac{193{,}500}{15{,}250} = 12.7$ sq.in. $\left[P = 16{,}000 - 70\dfrac{30}{2.78}\right.$ $\left. = 16{,}000 - 750 = 15250\right]$.

Therefore 4 angles $5 \times 3\frac{1}{2} \times 12(\frac{7}{16}) = 14.11$ sq.in. will be ample.

The addition of another pair of stiffeners over the center of the bearings would help greatly in distributing the pressure over the wall plates, as their outstanding



(f) (g)

FIG. 12.

legs would prevent an upward deflection of the plates and angles between the pairs of stiffeners. The additional stiffeners are shown by dotted lines in Fig. 12 (a) and (b).

A better bearing for the ends of this girder could be made by making it shorter in the direction of length of the girder and using either a cast or built-up pedestal to distribute the load in a transverse direction as is indicated in (*f*) and (*g*) of Fig. 12. For a length of 12″ the width should be 27″. The sizes for either case (*f*) or (*g*) should be determined from the laws of flexure, and direct stress.

For longer girders with greater end reactions to secure proper distribution of the load on the masonry and proper application of the reaction to the girder, a bearing should be used which by means of its form and action will insure this result. This is generally accomplished by means of an upper and lower shoe, both of which may freely rotate about a pin. These upper and lower shoes should have

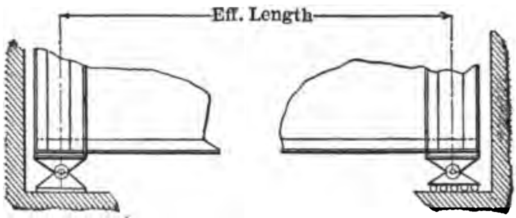


FIG. 12*h*.

proper bending strength and proper bearing on the pin and masonry. These shoes may be either built up of rolled material or cast in one piece. Their detailed design will not be undertaken here, as no new principles are to be developed.

PROBLEM

12. Design an end bearing for a girder of a single-track railway bridge of 36′ effective length, the cross-section of the girder at the

end being 1 web plate $48\times\frac{7}{16}$ and 4 angles $6\times6\times19.2(\frac{1}{2})$, the end reaction being 180,000 lbs.

APPENDIX

ART. 13. POSITIONS OF LOADING FOR MAXIMUM SHEAR AND MOMENT

The shears and moments, which are necessary for the design of plate girder bridges, are required for such a great variety of span lengths within the limiting lengths of span for such structures, that special methods of computing and tabulating them are advisable. In the principal Railway Engineers' Bridge Works and Consulting Engineers' offices the major part of the structures designed will be made for some standard loading. The loading known as Cooper E_{50} is perhaps more widely used than any other for railway bridges. The quantities given in Table No. 1 are for Cooper's E_{50} and are similar to those which should be determined and kept on record for any standard loading to enable design to be made with proper facility.

Shears

In computing shears it should be noted that for deck bridges the maximum end shear is given for the engine located so that either the first or last driving wheel stands at the end of the span.

The maximum shear at an interior point on the left of the center of the span is given when the load extends from the right and up to the point and perhaps a little beyond. Every load that passes from just to the right

of the point to just the left of the point under consideration causes a decrease in shear by the amount of that load. Any further movement of the system to the left increases the shear until another load passes from the right to the left of the point. The maximum shear must therefore be determined by trial and generally will occur for the first or second engine wheel just to the right of the given point.

For through bridges the maximum shear in any panel is given when the well-known criterion $P=\frac{W}{m}$ is satisfied, in which P is the load in the panel, W the total load on the bridge and m the number of equal panels in the span. Sometimes two or even three positions of the load satisfy the criterion, in which case the maximum shear is determined by comupting the shear for all positions which satisfy the criterion.

For maximum concentrated load brought to a floor beam or trestle bent by two adjacent stringers or girders: The criterion for position is $P=\frac{nW}{n+m}$, in which n and m are the lengths of the adjacent spans and P the load on the span of length n, and W the load on both spans.

Bending Moments

The bending moment, due to a certain number of moving loads at any definite point in a girder either deck or through, is a maximum when $P=\frac{nW}{m}$, in which P is the load on the left of the point, W the total load on the structure and n the distance from the point to the left

end and m the span length. This criterion enables any group of loads to be placed to produce a maximum moment. The criterion will sometimes be satisfied by more than one position of the live load, generally that position which has the heaviest load at the point under consideration and in addition the greater load on the structure is the one giving maximum moment. The moment for all positions of load which satisfies the criterion must be computed for a certain determination of the maximum.

For a deck structure the point of maximum bending moment occurs at or near the center, generally a little distance away from the center, the location of this point of maximum moment being different for different systems of loading. To aid in determining the position of the loading for absolute maximum it should be remembered that:

(*a*) The maximum moment must occur where the shear passes through zero;

(*b*) For a system of concentrated loads the shear must pass through zero at one of the loads;

(*c*) The amount of load on either segment, into which the point of maximum moment divides the span, is to the total load on the span as the length of the segment is to the span length.

These requirements for practical cases fix the point of maximum bending moment under one of the two wheels adjacent to the resultant of the system. The criterion for the exact position of the loading to produce maximum bending moment follows:

For any given system of loading, *the loads should be so placed that the center of the span is half way between the resultant of the system and one of the two loads which are nearest to the resultant.* This criterion may be established as follows:

Let Fig. 13a show any girder carrying the system of moving loads shown.

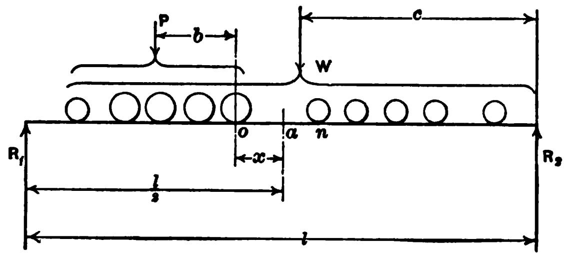


FIG. 13a.

Let a be the center of the span;

o be the point of maximum moment which occurs under o or n;

the resultant of the system $= W$;

c be the distance of W from the right end;

the resultant of the loads to the left of o be P;

b be the distance from o to P;

x be the distance from o to a;

Then the bending moment at

$$o = M = R_1\left(\frac{l}{2} - x\right) - Pb = \frac{Wc}{l}\left(\frac{l}{2} - x\right) - Pb$$

$$= \frac{Wc}{2} - \frac{Wcx}{l} - Pb. \quad . \quad (\alpha)$$

Now suppose the loading to advance a small distance to the left of dx, then $b=b$, $x=x+dx$, and $c=c+dx$, and the bending moment

$$M'=R_1\left(\frac{l}{2}-[x+dx]\right)-Pb \quad \text{as} \quad R_1=\frac{(Wc+dx)}{l} \text{ this}$$

$$=\frac{W(c+dx)}{l}\left(\frac{l}{2}-[x+dx]\right)-Pb;$$

neglecting terms containing dx^2,

$$M'=\frac{Wc}{2}-\frac{Wcx}{l}-\frac{Wcdx}{l}+\frac{Wdx}{2}-\frac{Wxdx}{l}-Pb. \quad . \quad (\beta)$$

Subtracting (α) from (β)

$$M-M'=dm=-\frac{Wdx}{2}+\frac{Wxds}{l}+\frac{Wcdx}{l}$$

$$\frac{dm}{dx}=-\frac{W}{2}+\frac{Wx}{l}+\frac{Wc}{l},$$

and for a maximum this must equal 0.

$$\therefore \quad -\frac{W}{2}+\frac{Wx}{l}+\frac{Wc}{l}=0$$

$$-\frac{l}{2}+x+c=0$$

and

$$x=\frac{l}{2}-c,$$

which establishes the criterion as stated.

PROBLEMS

13a. Compute the maximum bending moment for a deck plate girder of 18′ effective span length for Cooper's E_{50}. Assume the girder to carry one-half the loading.

13b. Compute the maximum bending moment for a 25′ span, same loading as 13a.

13c. Compute the maximum center moment for a 25′ span, same loading as 13a.

ART. 14. PREPARATION OF A TABLE OF BENDING MOMENTS, SHEARS, AND CONCENTRATED LOADS FOR COOPER'S E_{50} LOADING

The great advantage of a table of moments and shears is that it may be prepared in a few days for all spans for which it is at all likely to be needed. Computations for a system of quantities made at one time show by the law of the increase or decrease of the quantities any error, and the similarity of the computations enables them to be very rapidly made. When a system of many loads moves over spans of varying length it is evident that *one load* produces *maximum moment* for spans up to a certain length, two loads for a certain other length, and so on.

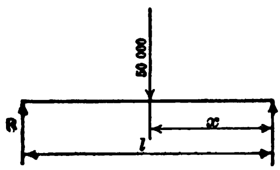


FIG. 14*a*.

For one load on a span see Fig. 14*a*.

$$M = R(l - x) = \frac{50,000x}{l}(l - x) = 50,000x - \frac{50,000x^2}{l}$$

$$\frac{dM}{dx} - 50,000 - \frac{100,000x}{l} = 0$$

or $\quad M = \text{a max., when} \qquad x = \frac{l}{2}.$

$\therefore\ M = \dfrac{50,000l}{4}$, which, as is well known, is the general expression for max. moment for any span with one load.

The general equation for moment for two loads spaced 5′ apart is

$$M = R(l - 5 - x) = \frac{250{,}000 + 100{,}000x}{l}(l - x - 5)$$

$$= 250{,}000 \times 100{,}000x - 750{,}000\frac{x}{l} - 100{,}000\frac{x^2}{l} - \frac{1{,}250{,}000}{l},$$

$$\frac{dM}{dx} = 100{,}000 - \frac{750{,}000}{l} - \frac{200{,}000x}{l} = 0. \quad \therefore\ x = \frac{l}{2} - 3.75.$$

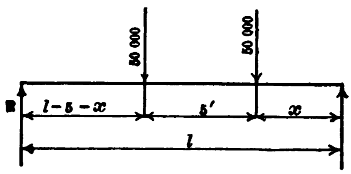


Fig. 14*b*.

Substituting for x its vlaue, we have for

$$M = \frac{250{,}000 + 100{,}000\left(\frac{l}{2} - 3.75\right)}{l}\left[l - \frac{l}{2} + 3.75 - 5\right]$$

$$= 250{,}000l - 125{,}000 + \frac{156{,}250}{l}.$$

By plotting the curves of moments for different lengths of spans we see that somewhere between 8′ and 9′ one load and two loads produce the same moment.

To find this point exactly we make the equations for moments simultaneous and solve to find the value of l for which the moments are equal.

For this case $\frac{50{,}000l}{4} = 250{,}000l - 125{,}000 + \frac{156{,}250}{l}$, whence $l = 8.54'$.

The next step is to get the general equation of moments for three loads. Make it simultaneous with that for two

loads, and solve for l, which gives the upper limit for two loads as 11.125′.

The equation for 4 loads is

$$M = 50{,}000l - 500{,}000 + \frac{312{,}500}{l}.$$

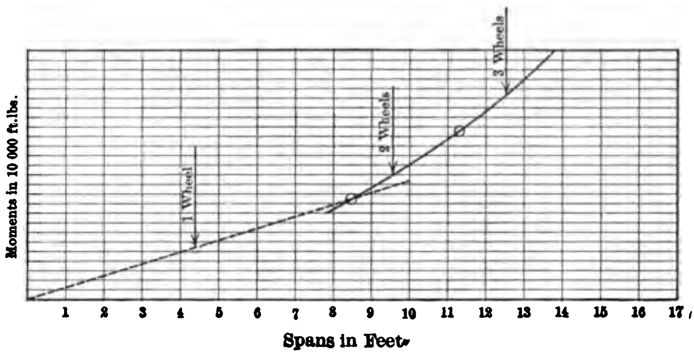


FIG. 14c.

The equation for five loads will now be written

$$\text{dist. of c.g. from right load} = \frac{2{,}075{,}000}{225{,}000} = 9.22'.$$

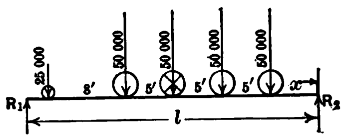


FIG. 14d.

The point of max. moment will be under the second driver.

$$\text{Reaction at } R_1 = \frac{2{,}075{,}000 + 225{,}000x}{l}$$

Maximum bending moment

$$=\frac{2{,}075{,}000+225{,}000x}{l}\times(l-x-23+13)-575{,}000$$

$$=\frac{2{,}075{,}000+225{,}000x}{l}\times(l-x-10)-575{,}000$$

$$=2{,}075{,}000+225{,}000x-\frac{2{,}075{,}000x+225{,}000x}{l}$$

$$-\frac{20{,}750{,}000+2{,}250{,}000x}{l}-575{,}000$$

$$\frac{dM}{dx}=225{,}000-\frac{2{,}075{,}000}{l}-\frac{450{,}000x}{l}-\frac{2{,}250{,}000}{l}=0$$

$$-\frac{450{,}000x}{l}=-225{,}000+\frac{2{,}075{,}000+2{,}250{,}000}{l}$$

$$450{,}000x=225{,}000l-4{,}325{,}000$$

$$x=\frac{l}{2}-9.61.$$

Substituting this value for x in the following simplified expression for the moment

$$1{,}500{,}000+225{,}000x-\frac{4{,}325{,}000x}{l}-\frac{225{,}000x^2}{l}-\frac{2{,}075{,}000}{l}$$

there is given

$$M=1{,}500{,}000+225{,}000\left(\frac{l}{2}-9.61\right)-\frac{4{,}325{,}000}{l}\left(\frac{l}{2}-9.61\right)$$

$$-\frac{225{,}000}{l}\left(\frac{l}{2}-9.61\right)^2-\frac{20{,}750{,}000}{l}$$

$$=1{,}500{,}000+112{,}500l-2{,}162{,}500-2{,}162{,}500+\frac{41{,}568{,}056}{l}$$

$$-56{,}250l+2{,}162{,}500-\frac{20{,}783{,}925}{l}-\frac{20{,}750{,}000}{l}$$

$$=-662{,}500+\frac{34{,}131}{l}+56{,}250l=56{,}250l-662{,}500+\frac{34{,}131}{l}.$$

Making the moments for four and five loads equal, we have:

$$50{,}000l - 500{,}000 + \frac{312{,}500}{l} = -662{,}500 + \frac{34{,}131}{l} + 56{,}250l$$

$$-6250l = -162{,}500 - \frac{278{,}369}{l}$$

$$6250l^2 - 162{,}500l = +278{,}369$$

$$l^2 - 26l = 44.54$$

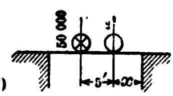
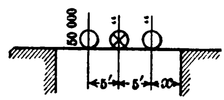
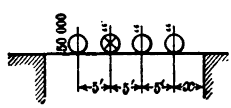
$$l^2 - 26l + 169 = 44.54 + 169$$

$$l - 13 = \pm 213.54 = \pm 14.61$$

$$l = 13 \pm 14.61 = 27.61'$$

as the limiting length of span for which four loads produce maximum moments.

The tabulation following shows the moments and the positions of the loads that produce then for the spans from 9 to 27′ for intervals of 1′.

Span =	Mom. =		
9	117,360	Limits: 8.536′ to 11.124′ $x = \frac{l}{2} - 3.75'$ $R = 50{,}000 - \frac{125{,}000}{l}$ $M = 25{,}000l - 125{,}000 + \frac{156{,}250}{l}$	
10	140,625		
11	164,200		
12	200,000		
13	237,500		
14	275,000		
15	312,500		
16	350,000	Limits: 11.125′ to 18.66′ $x = \frac{l}{2} - 5'$ $R = 75{,}000$ $M = 37{,}500l - 250{,}000$	
17	387,500		
18	425,000		
19	466,450		
20	515,625		
21	564,880		
22	614,210	Limits: 18.67′ to 27.61′ $x = \frac{l}{2} - 8.75'$ $R = 100{,}000 - \frac{250{,}000}{l}$ $M = 50{,}000l - 50{,}000 + \frac{312{,}500}{l}$	
23	663,500		
24	713,020		
25	762,500		
26	812,020		
27	861,570		
28			
29			
30			
31			
32			
33			
34			
35			
36			
37			
38			

PROBLEMS

14a. Compute the foregoing table up to 38′.

14b. Compute the maximum *ll* end shear for one girder of a 38′ effective single-track deck plate-girder bridge. Two girders carry one track. Cooper's E_{50}.

ART. 15. TABLE OF MOMENTS, SHEARS AND CONCENTRATED LOADS FOR COOPER'S E_{50}

The method of writing equations for bending moments of Art. 14 need not be followed for spans over 30′ long. For spans over 30′ in length the maximum center moment is only a small fraction of one per cent. less than the absolute maximum moment. The following table gives a convenient arrangement of the quantities for spans up to 75′ effective length.

For through spans tables of moments at the panel points and shears in all the panels for varying number of panels and panel length are readily prepared and should be made for any loading as much used as Cooper's systems.

PROBLEMS. Loading Cooper's E_{50}.

15a. Compute the live load concentration for a trestle bent which carries adjacent spans of 30′ and 60′. Show by a sketch the position of loads.

15b. Compute the maximum center moment for a 60′ span. Show by sketch the position of loads.

15c. Compute the maximum shear at the center of a 60′ span. Show by sketch the position of loads.

15d. Compute the maximum shear at a point 15′ from one end of a 60′ span. Show by sketch the position of the loads.

Span in ft.	End Sh.	Q. Pt. Sh.	C. Sh.	C. Mom.	Conc. Fl. Bm.	Span in ft.	End Sh.	Q. Pt. Sh.	C. Sh.	C. Mom.
10′	75.0	50.0	25.0	140.6	100.0	43′	198.4	122.1	57.3	1856.5
11′	81.9	52.3	27.3	164.3	109.1	44′	201.4	123.8	58.0	1929.0
12′	87.5	54.1	29.1	200.0	116.6	45′	204.3	125.4	58.6	2001.5
13′	92.3	55.8	30.8	237.5	123.1	46′	207.0	126.8	59.3	2074.0
14′	96.4	58.9	32.1	275.0	130.4	47′	209.9	128.5	59.9	2146.5
15′	100.0	62.5	33.4	312.5	136.6	48′	212.6	130.3	60.4	2219.0
16′	106.2	65.6	34.4	350.0	142.3	49′	215.4	132.0	61.3	2297.0
17′	111.8	68.4	35.3	387.5	147.0	50′	218.1	133.6	62.1	2377.3
18′	116.6	70.9	36.1	425.0	151.6	51′	220.9	135.3	63.0	2457.6
19′	121.0	73.0	36.0	466.5	157.3	52′	223.5	136.9	63.8	2538.0
20′	125.0	75.0	35.9	515.6	163.9	53′	226.3	138.4	64.5	2618.4
21′	128.6	78.6	37.0	564.9	169.9	54′	229.0	140.1	65.3	2702.6
22′	131.9	81.9	38.0	614.3	175.4	55′	231.8	142.0	66.0	2791.5
23′	134.8	84.7	38.9	663.6	180.4	56′	234.4	143.8	66.6	2880.1
24′	137.5	87.5	39.8	713.0	184.9	57′	237.1	145.5	67.3	2968.9
25′	142.0	90.0	40.5	762.5	189.1	58′	239.8	147.1	67.9	3057.6
26′	145.3	92.3	41.4	812.0	194.3	59′	242.4	148.8	68.8	3150.5
27′	148.1	94.5	42.1	861.6	200.3	60′	245.0	150.3	69.6	3247.1
28′	151.1	96.4	42.9	913.8	205.8	61′	247.8	151.8	70.5	3343.9
29′	153.9	98.3	43.5	969.9	210.9	62′	250.4	153.3	71.3	3440.5
30′	157.6	100.0	44.1	1026.1	215.6	63′	253.6	154.8	72.0	3537.3
31′	161.1	101.9	45.5	1082.3		64′	256.8	156.3	72.8	3638.8
32′	164.3	103.9	46.9	1138.6		65′	259.5	157.8	73.5	3743.8
33′	167.4	105.9	48.1	1194.9		66′	262.4	159.3	74.3	3848.8
34′	170.3	107.8	49.3	1251.0		67′	266.0	160.8	74.9	3963.8
35′	172.9	109.5	50.4	1307.4		68′	269.5	162.3	75.6	4058.8
36′	176.4	111.1	51.4	1371.7		69′	273.0	163.8	76.2	4163.8
37′	179.6	112.5	52.4	1435.9		70′	276.3	165.3	76.9	4268.8
38′	182.8	113.8	53.3	1500.0		71′	279.5	166.8	77.7	4376.0
39′	185.8	115.0	54.1	1566.6		72′	283.4	168.3	78.5	4481.3
40′	188.5	116.9	55.0	1639.1		73′	287.0	169.9	79.2	4588.1
41′	192.0	118.8	55.8	1711.6		74′	290.6	171.3	80.0	4700.0
42′	195.3	120.5	56.5	1784.1		75′	294.3	172.6	80.7	4813.8

Fl. Bm. Conc. 30′ and 60′ = 298.6 40′ and 60′ = 328.6

Note: Shears in thousands of lbs. and Moments in thousands of ft.-lbs. Shears and Moments for one track.